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MODELING INTERACTIVE EFFECTS IN MATHEMATICAL FUNCTIONS FOR BUSINESS SIMULATIONS: A CRITIQUE OF GOOSEN'S INTERPOLATION APPROACH

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ABSTRACT

Goosen (1986) presented an *interpolation approach* as a simplified and effective way to model complex and non-linear demand and cost functions in computerized business simulations. However the usefulness of the approach becomes limited when one considers the possibility of *interactive effects* between the dependent and independent variables which are embodied in most business relationships. The important issue of interactive effects are not directly addressed in Goosen's interpolation method and it is not clear how these types of effects would be systematically included in the general case. The potential usefulness and application of the interpolation method depends on whether or not it could directly and efficiently address the procedures necessary to incorporate interactive effects.

PURPOSE

A paper by Goosen in the 1 986 issue of the ABSEL Proceedings, titled "An Interpolation Approach To Developing Mathematical Functions For Business Simulations" presents a graphically oriented algorithm for designing functional relationships. Goosen (1986) argues that his interpolation approach is easy to implement and capable of generating results almost identical to more complex curvilinear functions, like those developed by Gold & Pray (1984,1989,1992), Thavikulwat (1989), and Teach (1990) for modeling demand and production relationships. The purpose of this paper is to identify an important issue, namely *interactive effects,* that need to be more carefully addressed in the "interpolation approach" of Goosen before it can be generally applied to model sophisticated demand, production, and cost relationships in business simulations.

INTERACTIVE EFFECTS

Sophisticated demand, production, or cost functions must allow for interactive effects to exist between the independent and dependent variables. An interactive effect exists when the relationship between two variables, say Q and X, is influenced by the level of a third variable, say Y. These interactive effects may be linear but, more generally, are non-linear.

Interactive effects may be expressed mathematically given a generalized two variable demand function for illustrative purposes:
An interactive effect exists between price (P1 and advertising (A) if

$$Q = f(P,A) \qquad (1)$$

where:
Q = quantity (units/period)
P = price ($/unit)
A = advertizing ($/period)

the partial derivative of Q with respect to P changes with the level of the variable A, that is:

$$\frac{d(dQ/dP)}{d(A)} > 0 \qquad (2)$$

and/or the partial derivative of Q with respect to A changes with the level of the variable P:

$$\frac{d(dQ/dA)}{d(P)} > 0 \qquad (3)$$

No interactive effect exists if the partial derivatives in both equations 2 and 3 are equal to zero.

Interactive effects may also be expressed graphically by noting that the demand function line (or curve) which plots the relationship between price and quantity, assuming advertising expenditures are fixed, shifts to the right as the level of `advertising increases. Similarly, the demand function line or curve which plots the relationship between advertising and quantity, assuming price is fixed, shifts to the left as the price is increased.

INTERPOLATION METHOD

The five-step interpolation approach outlined by Goosen (1986) does not directly address how interactive effects can be modeled. *Step one* of the interpolation approach is to sketch on graph paper the desired curvilinear functional relationship between two variables, like price and quantity. (Shifts in the functional relationship are not discussed.) Based on the graph in step one, a set of points on the function are selected *(step two),* and a schedule is prepared listing the values assigned to the independent variable and the resulting values for the dependent variable *(step* an interpolation equation is then developed and programmed *(steps four and five 4)*. Goosen's interpolation equation is:

$$IV = Y(i) + \frac{DV - X(i)}{X(i+1) - X(i)} [Y(i+1)] \qquad (4)$$

where: IV = interpolated value for dependent variable Y
Y(i) = dependent variable value at point i
X(i) = independent variable value at point i
DV = decision value for independent variable X

In equation (4), the interpolated value (IV) depends on the plotted values for X (i) and Y(i) which are read from the graph of the function developed in steps 2 and 3 of the interpolation method. The values for Y (i) only depend on the values of X (i) in this methodology. Consequently the interpolated value for the variable V does not directly account for or incorporate the possibility of interactive effects (like those described previously between price and advertising).

The modeling of interactive effects would require the graph developed in step 1 of the interpolation method to shift to the right (or left) as the value of an "external" independent variable, like Z (i), were to change. How this type of phenomenon would be integrated into Goosen's interpolation algorithm is not clear.

INTERPOLATION EXAMPLE WITH GOLD-PRAY FUNCTION

Goosen provided an example using the interpolation method to emulate the demand function developed by Gold & Pray (1984). The Gold-Pray demand function is a generalized multiplicative market demand model, which incorporates *interactive effects* between the independent variables. In the example the interpolation method was shown to closely approximate the Gold-Pray function. The interpolation approach was argued to be a simpler and, by implication, a more efficient method for modeling complex functional relationships.

The interesting question is how the two methods could be comparable when the interpolation approach does not directly incorporate interactive effects? The answer is that in the example developed by Goosen an interactive effect was added, implicitly, to the interpolation method. The author modified his own interpolation equation as follows:

$$IV^* = IV \times [1 + IEF] \qquad (5)$$

In equation 5 the original interpolated value (IV) is multiplied by

where:

IV* = adjusted interpolated value
IV = original interpolated value (defined in equation 4 above)
IEF = interactive effect factor

an interactive effect factor to calculate an "adjusted" interpolated value that now includes an interactive effect. In the example by Goosen the interactive effect factor (IEF) was given as 606.63% and was derived from the Gold-Pray demand function.

The modified approach seems simple enough but becomes more complex when carefully evaluated. an interpolation approach would be needed to obtain the interactive effect factor (IEF), i.e. one would have to go through steps 1 to 4 outlined by Goosen just to calculate the IEF before beginning to calculate IV*. Second, their may be more than one interactive effect factor, which is the general case for most business simulations. For example, market demand depends on a set of variables like: substitute price, rival advertising and promotion campaigns, interest rates, sales force, economic growth, inflation rates, and research and development expenditures (to name just a few). All of these factors are highly interrelated and require interactive effects to be modeled in the demand algorithm. Given several interactive effects, the interpolation approach by Goosen would become much more difficult to apply. Third, the interactive effect factor my change with the level of the other independent variables. In the Gold-Pray function the IEF factor for advertising of 606.63% was independent of the level of price. This may not be the desired case in some business simulations. If the advertising elasticity of the firm were presumed to change with the price level, then the IEF factor would change even if advertising expenditures were held constant. This would further complicate the interpolation approach by Goosen.

SUMMARY

Goosen's interpolation approach is presented as a simplified approach to modeling complex and non-linear functional forms. However the usefulness of the approach becomes limited when one considers the possibility of interactive effects between the dependent and independent variables in the functional relationships embodied in most business simulations. Interactive effects are not directly addressed in Goosen's interpolation method and it is not clear how these type of effects would be systematically included in the general case. The potential usefulness and application of the interpolation method depends on whether or not it could directly and efficiently address the procedures necessary to incorporate interactive effects.

REFERENCES

Gold, S. and Pray, T. (1984), "Modeling Market and Firm Level Demand Functions in Computerized Business Simulations", Simulation & Games: An International Journal of Theory. Design and Research, Vol. 15, No. 3, pp. 346 - 363

Gold, S. and Pray, T. (1989), "The Production Frontier: Modeling Production in Computerized Business Simulations", Simulation & Games: An International Journal of Theory. Design and Research, Vol. 20, No. 3, pp. 300- 318

Gold, S. and Pray, T. (1 992), Modeling Short-Run Cost and Production Functions in Computerized Business Simulations", Simulation & Games: An International Journal of Theory, Design and Research, forthcoming Vol. 23, No. 4, pp. 41 7 - 430.

Goosen, K., (1986), "An Interpolation Approach to Developing Mathematical Functions for Business Simulations', Developments in Business Simulations & Experiential Exercises, Vol. 13, pp. 248 -255.

Teach, R. D. (1990), 'Demand Equations which include Product Attributes", Developments in Business Simulations & Experiential Exercises Vol. 17, pp. 161 - 166.

Thavikulwat, P. (1989), "Consumption as the Objective in Computer-Scored Total Enterprise Simulations", Developments in Business Simulations & Experiential Exercises, Vol. 16, pp. 37-40.