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A LINEAR PROGRAMMING APPROACH TO OPEN SYSTEM TOTAL ENTERPRISE SIMULATIONS

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ABSTRACT

The promise of open system simulations is no longer a promise. One method of devising them is to use the goal oriented version of linear programming with a focus on the reverse assumption of the standard economic theory of industrial organization. That is, the Structure→Process→Performance model of economic theory needs to be reversed, at east, to Process→Structure→Performance. Doing this eliminates the need for and fixed algorithms that drive current total enterprise (TE) simulations. More important, this sort of logic suggests an even more general approach characterized by a completely interactive structure, process, and performance model.

INTRODUCTION

Without exception, all popular total enterprise (TE) simulations adhere to the economic theory of industrial organization. The authors of these games pay particular attention to the basic assumptions that (a) the structure of an industry determines the processes that occur in that industry, and (b) the processes in an industry determine its resource allocation efficiency or performance (Patz, 1981). In symbols, these two premises may be represented by a simple linear heuristic: Structure→ Process→ Performance.

Key structural factors such as concentration, product differentiation, scale economies, price elasticity of demand, entry conditions, and demand growth and decay are determined a priori, before any competition begins in the classroom. For example, one question is paramount when an administrator configures a TE competition: How many firms are in the industry? The answer given to this question, along with the overall demand growth and decay function determines total market size.

Some simulations permit market entry for additional firms as well as exit and mergers, but the overall effect is to create an oligopoly with a limited range of market share possibilities. Individual firms have fixed demand functions that depend upon such factors as price, advertising and R & D expenses, number of salespeople and distribution centers, and so forth-- usually with a highly imaginative set of lead, lag, average, and cumulative effects. The point is that these a priori, overall, and fixed functions remain the same throughout the competition, more or less stabilizing the price elasticity of demand.

Likewise, scale economies are fixed, once again, by usually creative production functions, and the net effect is a closed system (Patz, 1990). Indeed, these closed systems follow standard economic theory. However, it is not necessary to do so, and the twofold purpose of this paper is to show that routine linear programming theory can (a) generate open system TE simulations, and (b) stimulate further research on the nature of microeconomic activity.

REVERSING STANDARD ECONOMIC THEORY

Closed systems must result from standard economic theory. Once the structure of a market is fixed, everything else has to occur within its basic framework. Structure→Process→Performance does not leave much room for the "imperfections" exhibited in real markets. Economists and financial theorists attribute these quirks to systemic failures, that is, practice should conform to theory rather than vice versa.

But, if the standard theory is reversed, then some of these so-called imperfections may not be abnormal at all. Beginning with the reverse

assumption that Process→Structure→Performance leads to an entirely different way of looking at markets and TE simulations. There is evidence for taking this position (Winsor, 1989) and five representative processes will be used in this paper: pricing, promotion, quality, training, and automation represented as PR, PM, QY, TR, and AT.

The Price Constraint

For purposes of brevity, only the result of this linear programming approach will be presented. The meandering path that led its development is not worth recounting in such a brief presentation.

Therefore, take the pricing process and write a typical linear programming constraint (Dantzig, 1963) such as:

$$a_{11}PR_1 + \dots + a_{1n}PR_n \leq b_1 \quad (1)$$

where a_{1j} = a variable coefficient
 j = (1, ...n)
 n = a variable number of firms in the industry
 PR_j = firm j 's pricing decision for the current period,

and

$$b_1 = \text{a variable stipulation.}$$

Notice that price times quantity in equation (2) for b_1 is total market revenue. Likewise, making the appropriate substitutions from system (4) definitions into constraint (1):

$$b_1 = PR_{av}Q \quad (2)$$

where PR_{av} = some sort of average price, and
 Q = total market demand.

Elaborating further on Q , it could be a fairly complex function such as

$$Q = F(1/Pr_{av}, PM_{av}, QY_{av}, TR_{av}, AT_{av}) \quad (3)$$

where the various process averages, such as QY_{av} , may take many forms. In short, the immediate purpose is not to argue specific forms but to demonstrate the feasibility of a linear programming formulation.

Now, looking at the variables on the left-hand side of equation (1), let

$$a_{11} = f_{11}Q = \text{Firm 1 unit sales} \quad (4)$$

$$a_{1n} = f_{1n}Q = \text{Firm n unitsales}$$

such that

$$\sum_{1j} \leq 1, \quad (5)$$

and $f_{1j} \geq 0$. Only if $\sum_{1j} = 1$ does the first constraint (1) equality condition hold

that $a_{11}PR_1 + \dots + a_{1n}PR_n = b_1$.

Notice that price times quantity in equation (2) for b_1 is total market revenue.

Likewise, making the appropriate substitutions from system (4) definitions into constraint (1):

$$f_{11}(Q)PR_1 + \dots + f_{1n}(Q)PR_n \leq PR_{av}(Q) \quad (6)$$

where $f_{1n}(Q)PR_{11}$ is Firm 1's revenue and so forth. Canceling Q in (6) yields

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$$f_{11}PR_1 + \dots + f_{1n}PR_n \leq Pr_{av} \quad (7)$$

with the condition (5) still holding that $\sum f_{ij} < 1$. Since the PR_j 's are constant in any given period, the constraint (7) can be rewritten in the usual fashion:

$$PR_1 f_{11} + \dots + PR_n f_{1n} \leq Pr_{av} \quad (8)$$

Other Process Constraints and Relative Attractiveness

A similar development would lead to parallel constraints for the other four process variables--PM, QY, TR, and AT. For example, the promotion constraint would be

$$PM_1 g_{11} + \dots + PM_n g_{1n} \leq PM_{av} \quad (9)$$

Where it is not necessarily true that

$$f_{1j} = g_{1j} \quad (10)$$

But, assuming for the moment that the equality (10) holds, then the following system results:

$$\begin{aligned} PR_1 f_{11} + \dots + PR_n f_{1n} &\leq Pr_{av} \\ PM_1 g_{11} + \dots + PM_n g_{1n} &\leq PM_{av} \\ QY_1 f_{11} + \dots + QY_n f_{1n} &\leq QY_{av} \\ TR_1 f_{11} + \dots + TR_n f_{1n} &\leq TR_{av} \\ AT_1 f_{11} + \dots + AT_n f_{1n} &\leq AT_{av} \end{aligned} \quad (11)$$

Obviously, the second subscript on each f_{ij} is superfluous. Therefore, rewrite (11) as

$$\begin{aligned} PR_1 r_1 + \dots + PR_n r_n &\leq Pr_{av} \\ PM_1 r_1 + \dots + PM_n r_n &\leq PM_{av} \\ QY_1 r_1 + \dots + QY_n r_n &\leq QY_{av} \\ TR_1 r_1 + \dots + TR_n r_n &\leq TR_{av} \\ AT_1 r_1 + \dots + AT_n r_n &\leq AT_{av} \end{aligned} \quad (12)$$

where the determination of each element in the $(PR_{av}, \dots, AT_{av})T$ constraint vector, with T as the transpose notation, is similar to the problem posed by equation (3) for Q. That is, feasibility is the issue--not a final design.

Now, without immediate explanation, let

$$\begin{aligned} r_j &= \text{the relative attractiveness of firm } j \text{ to consumers} \\ k &= \text{the number of competing TE simulation teams,} \end{aligned}$$

say, in one section of a business policy course, where $k < n$. Then, as before, with $n = a$ variable number of firms in the industry ($n > k$), $x = n - k$ is the number of firms under administrator control. The variable x , unknown to the participating teams, can vary from zero to $(n-k)$ in any period at the discretion of the administrator.

The first key result of this development is that the administrator sets the relative attractiveness of $x = n - k$ firms by adding further constraints to the system (12). This can be done in many ways, but one overall constraint will hold. That is,

$$r_1 + \dots + r_n \leq 1 \quad (13)$$

since whatever initial values are assigned to the they can always be normalized such that (13) holds.

Then, for the $x = n - k$ administrator firms, some other constraint where a 1 will be needed. Again, the number of administrator firms may vary from period to period.

$$R_{k+1} + \dots + r_n \leq \alpha \quad (13)$$

Where $\alpha \leq 1$ will be needed. Again, the number of the administrator firms may vary from period to period.

Relative Attractiveness Revisited

However, back to relative attractiveness, r_j . What does this mean? What concepts support this notion?

System (12), actually, is the basic answer to these questions. Its proposition is that a firm j 's Process vector, represented simply here as

$$(PR_j, \dots, AT_j)^2 \quad (14)$$

determines its appeal to consumers based upon the process vectors of all other firms. These appeals noted as r_j , in turn, are equal to f_{ij} in system (11), where $f_{ij}(Q) = \text{Firm } j\text{'s unit sales in system (4)}$.

The main reason, of course, that each process vector determines firm j 's appeal to consumers, based upon the process vectors of all other is that they are dependent upon each other through system (12) and constraints (13) and (14). System (12) and constraints (13) and (14), in addition, make possible an open system TE simulation with (a) a variable number of firms in the industry, (b) relative attractiveness variations made possible through the goal programming extension of linear programming (Ijiri, 1965), and (c) a new and empirically useful way to view the imperfections of standard economic theory.

A GOAL PROGRAMMING SOLUTION

The variable number of firms has already been established in equations (13) and (14). What is needed is some sort of analytical solution to this "relative-attractiveness" concept. The new, empirical approach to standard economic theory will be considered in a subsequent section.

Stated in another way, regarding relative attractiveness, the problem focus is now on how to optimize something associated with system (12) and equations (13) and (14). Therefore, taking another look at this formulation,

$$\begin{aligned} PR_1 r_1 + \dots + PR_n r_n &\leq Pr_{av} \\ PM_1 r_1 + \dots + PM_n r_n &\leq PM_{av} \\ QY_1 r_1 + \dots + QY_n r_n &\leq QY_{av} \\ AT_1 r_1 + \dots + AT_n r_n &\leq AT_{av} \\ r_1 + \dots + r_n &\leq 1 \\ r_{k+1} + \dots + r_n &\leq \alpha \end{aligned} \quad (16)$$

it is clear that other constraints on linear combinations of the r_j 's could be added at the administrator's discretion. For example, specific values could be assigned for the relative attractiveness of each administrator firm in such a fashion that their sum equals α .

In fact, imagination is the only limit on the variations of system (16) that may be attained by adding or removing constraints on the terms. The key issues are how to how to solve the resulting system, whatever it is, and how to modify a system that has no solutions.

Solvable Systems

Assuming that system (16) is solvable in the form chosen by the administrator, the simplest solution procedure is the one provided by goal programming. That is, rewrite (16) as:

$$\begin{aligned} \text{Minimize } & Y_{PR}^+ + Y_{PR}^- + Y_{PM}^+ + Y_{PM}^- + \dots + Y_{AT}^+ + Y_{AT}^- \\ 2PR_1 r_1 + \dots + PR_n r_n - Y_{PR}^+ + Y_{PR}^- &= PR_{av} \\ PM_1 r_1 + \dots + PM_n r_n - Y_{PM}^+ + Y_{PM}^- &= PM_{av} \\ \dots & \\ AT_1 r_1 + \dots + AT_n r_n - Y_{AT}^+ + Y_{AT}^- &= AT_{av} \\ r_1 + \dots + r_n &\leq 1 \\ r_{k+1} + \dots + r_n &\leq \alpha \\ r_j &\geq 0 \\ Y_m^+, Y_m^- &\geq 0 \end{aligned} \quad (17)$$

where the y_m^+ 's and y_m^- 's, for $m = PR, PM, QY, TR, \text{ and } AT$, are overages and underages respectively for each process constraint.

For example, continuing the assumption that (17) has a solution and using the price constraint $PR_1 r_1 + \dots + PR_n r_n - Y_{PR}^+ + Y_{PR}^- = PR_{av}$, Y_{PR}^+ measures the amount by which $PR_1 r_1 + \dots + PR_n r_n$ exceeds PR_{av} in that solution. Likewise, Y_{PR}^- measures the amount by which it falls short. Furthermore, when viewing (17) as a matrix, the columns for y_{pr}^+ and y_{pr}^- are linearly dependent. One is simply the negative of the other. Therefore, only one can have a value different from zero in any

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solution.

A similar interpretation holds for every other process constraint in this example, or for any other process constraint that a TE simulation designer chooses to include. In short, the procedure is to minimize deviations from the augmented constraint vector $(PR_{av}, AT_{av}, 1, a)^T$, and it has two basic advantages. First, it does not impose a predefined structure on the market. Each firm's pricing, promotion, and so forth decisions do that. Second, it's simple. There is no need to classify unusual results as imperfections. What happens simply happens.

Unsolvable Systems

But, when system (17) does not have a solution, the routine simplex algorithm for solving linear programming problems will not suffice. An extended one is needed that halts execution when a problem is encountered alerts the administrator regarding which constraint is incompatible with a solution, and in more elaborate models, suggests modifications to the original problem.

This sort of extended simplex algorithm is entirely possible (Cooper, 1991), and its implementation involves basically a more complex computer routine. The theoretical Point of such a routine is that whatever the administrator does to resolve an unsolvable system the market. These market-clearing corrections, in turn, suggest new avenues of empirical research.

STRUCTURE VS. PROCESS: ONE MORE TIME

Dorfman, Samuelson and Solow (1958), of course, were among the first to note the advantages of linear programming over conventional mathematics in the development of economic theory. The preceding argument is just another example of their basic approach, but it adds several new dimensions by reversing the traditional notion of Structure→Process.

First, by focusing on process, a TE simulation of an industry becomes an open system. Any number of firms may enter or leave at the discretion of the administrator. Second, a firm's relative attractiveness to consumers is a composite of all its processes in relation to the processes of all competitors. This is also the case in traditional TE simulations, but the linear programming model allows for additional constraints on relative attractiveness per se. Third, the process focus in the linear programming system (17) is infinitely expandable. Unlike the a priori, and fixed algorithms in TE simulations, an administrator can vary the number and type of constraints in system (17) at will. Fourth, and most important, this approach adds at least two new research directions--market models and expert systems.

Market Models

As noted above in the case of unsolvable systems, whatever an administrator does to resolve the issue clears the market. In other words, imperfections are no longer imperfections when viewed in this manner. Markets simply clear one way or another. As constraints are modified in order to reach a solution, the inner workings of the market being created by process decisions become apparent.

The question to be answered is straightforward: What had to be done in any period in order to clear the market, solve the system? In fact, competing teams in a classroom are not required to answer this question. They would not qualify as a controlled experiment. Instead, specific decision patterns would have to be tested on plausible versions of system (17) in order to determine fundamental answers to the preceding question.

System (17) by itself, however, has to be expanded to include issues of production or service capacity, inventories, distribution, balance sheets, income statements, cash flows, and so forth. The five processes used for expository purposes in this paper are only a beginning. In fact, the assumption in equation (10) that equated the coefficients for each process variable is only a beginning. There is no reason to continue with this assumption beyond the purposes of brevity in this paper.

Nevertheless, experimental trial and error runs of system (17), using a flexible simplex algorithm (Erikson & Hall, 199x), have shown that it is a feasible approach. When solutions fail, reasonable guesses on where to begin again produce the needed market share or relative attractiveness answers so crucial to this development (Patz, 1991).

Expert Systems

Reasonable guesses, however, are not very elegant. What is needed, to repeat, is a simplex algorithm that provides solution suggestions.

This is where rule based linear programming, expert systems and TE simulations meet. The process has six steps:

- 1 - The expert system is the overall control program and begins by asking for decision inputs, as usual.
2. Decision inputs are placed into some version of a model such as system (17).
3. A simplex algorithm is invoked to solve the system.
4. Under no solution conditions, the expert system examines the situation and recommends solutions to the administrator r.
5. The administrator selects a solution based upon research or teaching goals.
6. Results are printed including income statements, balance sheets and cash flows.

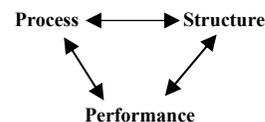
One of the key challenges in this process is to develop the knowledge base for the expert system that recommends the modifications necessary to achieve a solution to whatever model the administrator chooses. But basic linear programming is so well understood that this should be more of a routine than creative task. In fact, such knowledge bases may already be available.

If they are, they pose another interesting research problem. What market result differences do different combinations of rule based expert systems and knowledge bases create when applied to the same TE simulations? Once again, the research potential of open system TE simulations is boundless (Patz, 1987).

CONCLUSIONS

This last statement, more than anything else summarizes the twofold purpose of this paper. That is, open system TE simulations are possible, using a linear programming framework in this case. Second, they do cause a rethinking of economic theory and market or industry research. In this case, the standard Structure→Process→Performance model has been called into question.

The **Process→Structure→Performance** model relaxes the need for a overall, and fixed algorithms in TE simulations. Moreover, going beyond the arguments in this paper, this model suggests that a more general requirement is for TE simulations based upon the following model:



The simplicity of standard approaches will not suffice for the complex educational and business practices of international economies. Process, structure, and performance interact. They are not arranged in a simple causal fashion. A simple reversal of this linear model emphasizes the point.

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