USING THE LOGISTIC FUNCTION TO SCORE COMPUTER-SUPPORTED GAMES

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ABSTRACT

The Logistic function will normalize a distribution skewed towards the high end. Such a distribution is characteristic of achievement, as distinguished from ability. The use of the logistic function to score a genotypical, computer-supported game, Boss, on a periodic and cumulative basis is discussed. Because the logistic function is simple, bounded by two horizontal asymptotes, symmetric about the origin, and concave above the ordinate, it is ideally suited for scoring computer-supported games that approach everyday reality.

INTRODUCTION

Psychologists have observed that human abilities are normally distributed over the human population. Economists have observed, however, that the distribution of achievement, as measured by wealth and income, is skewed towards the high end. Mincer [1] reviews many explanations *of* this skewness, but regardless of the reasons, the skewness of achievement towards the high end is a fact that any business game sufficiently close to everyday reality should exhibit. This creates a problem, because students playing business games expect their grades to follow a normal distribution, or even one skewed towards the low end of the distribution. But if the game is close to everyday reality, then the distribution of achievement will be skewed, even in the ‘wrong direction.

Any monotonically-increasing concave function will tend to normalize a distribution skewed towards the high end. Economists often use the logarithmic function, but for the purpose of scoring a business game, a related function, the logistic [2], is especially advantageous. The logistic function of the form passes through and is symmetric about the origin. This function is completely determined by two

$$y = \frac{A}{1 + e^{-Abx}} - \frac{A}{2} \qquad (1)$$

parameters: A, the vertical distance between the two horizontal asymptotes; and b, a measure of the rate of change of with respect to x. Equivalently, of course, the function is completely determined by two points on its curve, or by one parameter and one point.

APPLICATION

To demonstrate how the logistic function can be used to score a business game, we shall describe one particular application. In this application, the game used was Boss [3], a computer-supported line-management game created by the author.

The Game

Boss is a game where making money is the sole objective. Money is made primarily by buying cheap and selling dear, and secondarily, by earning interest on money accumulated from the profits of trade. Although the game is structurally simple, it is not easy. The problems of forecasting price movements, of deciding how much and at what prices to buy, how much and at what prices to sell, and how long to hold inventory constitute one level of difficulty. And because the game allows a sizable advantage to players who, instead of making their own decisions, employ other players to make decisions for them, problems of human relations constitute a second level of difficulty.

The game is supported by a set of interactive computer programs allowing for convenient access, immediate execution of decisions, and immediate confirmation of results. These programs do not simulate a market, rather, they facilitate a true market, for the game is designed such that players buy and sell with one other, and not with the computer. The computer, however, lists the offers and counteroffers, and it executes the trades. In essence, the game is structurally real. It is not just phenotypical of reality; it is genotypical to reality.

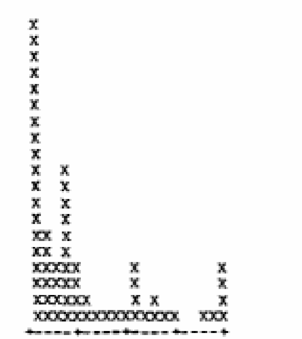
Because the game has a singular criterion of success, money, no arbitrary weighing scheme is necessary to compute scores. The only arbitrariness is in the question of when the game is to be scored. Usually, the game is scored weekly over the semester the game is played, the scoring accomplished by giving points in exchange for money that players turn in. Players decide the amount of money they will turn in, and the amount they will keep to use in continuing the game. The points they get in return are determined by a logistic function, which gives progressively diminishing returns of points for money.

Data

The game was used in conjunction with two sections of an undergraduate class on business policy at a major state university in the Midwestern United States. Data from a one-semester, 12-week run of the game were collected. These are given below to illustrate how the logistic function was used to score the Lame.

The game was scored weekly. Figure 1 is a histogram showing the distribution of money submitted for points on the second week. The base of the histogram is given

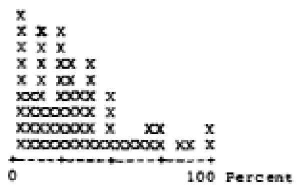
FIGURE 1
DISTRIBUTION OF MONEY SUBMITTED FOR POINTS, SECOND WEEK



Note: Each X represents one player.

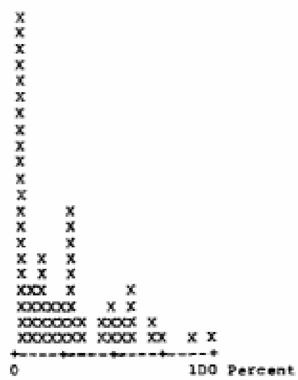
in percentage points, 100 percent representing the highest amount of money submitted. Figures 2 and *3* show similar histograms on the sixth and tenth week, respectively. As for the money players retain in their businesses, histograms showing these corresponding distributions are given in Figures 4, 5, and 6.

FIGURE 2
DISTRIBUTION OF MONEY SUBMITTED FOR POINTS, SIXTH WEEK



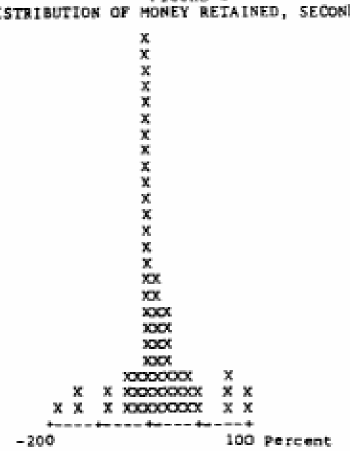
Note: Each X represents one player.

FIGURE 3
DISTRIBUTION OF MONEY SUBMITTED FOR POINTS, TENTH WEEK



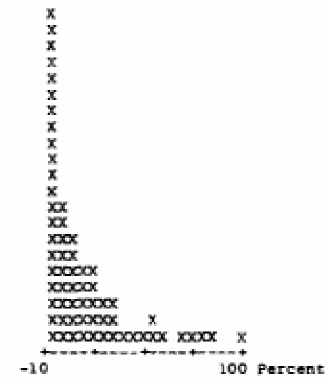
Note: Each X represents one player.

FIGURE 4
DISTRIBUTION OF MONEY RETAINED, SECOND WEEK



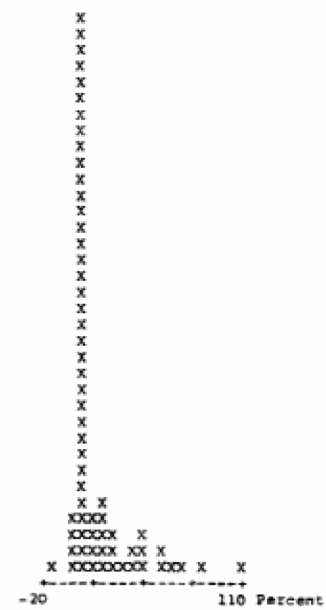
Note: Each X represents one player.

FIGURE 5
DISTRIBUTION OF MONEY RETAINED, SIXTH WEEK



Note: Each X represents one player.

FIGURE 6
DISTRIBUTION OF MONEY RETAINED, TENTH WEEK



Note: Each X represents one player.

If money truly measures achievement in this Lame, then theory predicts the distribution of money shall be skewed towards the high end, with the skewness being more pronounced as the game progresses. This is true in the distributions obtained. The distributions of money submitted are clearly skewed towards the high end as early as the second week of the game. The distributions of money retained are similarly skewed by the sixth week. Thus, the game's sole criterion of money must truly measure achievement in the game.

Money was converted to points by a logistic function that defined the distance separating the asymptotes to be 4 points, and the highest, or 100 percent score, to be 1.96 points. Thus,

$$y = \frac{4}{1 + e^{-4bx}} - 2 \qquad (2)$$

and

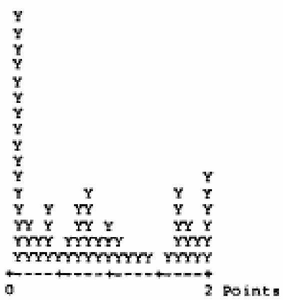
$$1.96 = \frac{4}{1 + e^{-4b}} - 2. \tag{3}$$

These two equations reduced to

$$y = \frac{4}{1 + e^{-4.595x}} - 2. \tag{4}$$

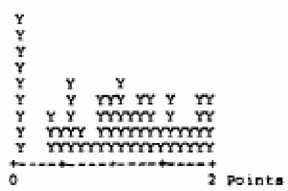
The distributions of point scores corresponding to the distributions of money submitted are given in Figures 7, 8, and 9. These transformed distributions are more normal, as they must be.

FIGURE 7
DISTRIBUTION OF POINT SCORES, SECOND WEEK



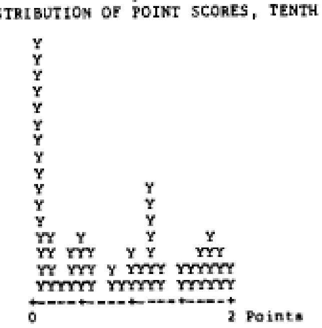
Note: Each Y represents one player.

FIGURE 8
DISTRIBUTION OF POINT SCORES, SIXTH WEEK



Note: Each Y represents one player.

FIGURE 9
DISTRIBUTION OF POINT SCORES, TENTH WEEK



Note: Each Y represents one player.

One problem remained, that of scoring the game when it ended. To be fair, money submitted periodically and money left until the end should have earned approximately the same number of points. That is, the time players chose to turn in money for points should not have unduly affected their scores.

We solved this problem by defining the logistic function transforming remaining money into points such that the functions asymptote was the periodic asymptote multiplied by the number of periodic scoring times (2 multiplied by 11, equaling 22), such that the amount of money that constituted a 100-percent score was the average of the present values of the amounts that had constituted such a score in all previous scoring times, and such that the point score at 100 percent was the same as the 100-percent periodic point score (1.96). Thus,

The distribution of money left at the end of the game is shown in Figure 10, and the periodic and final scoring functions are plotted together in Figure 11.

$$y = \frac{44}{1 + e^{-4bx}} - 22 \tag{5}$$

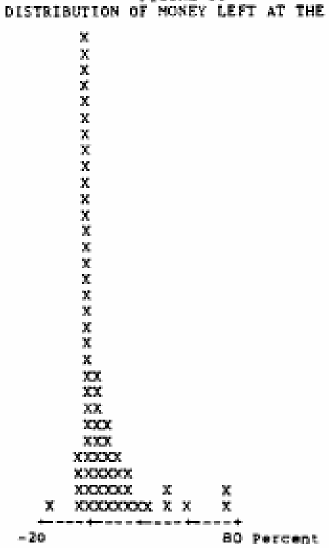
and

$$1.96 = \frac{44}{1 + e^{-4bx}} - 22. \tag{6}$$

These two equations reduced to

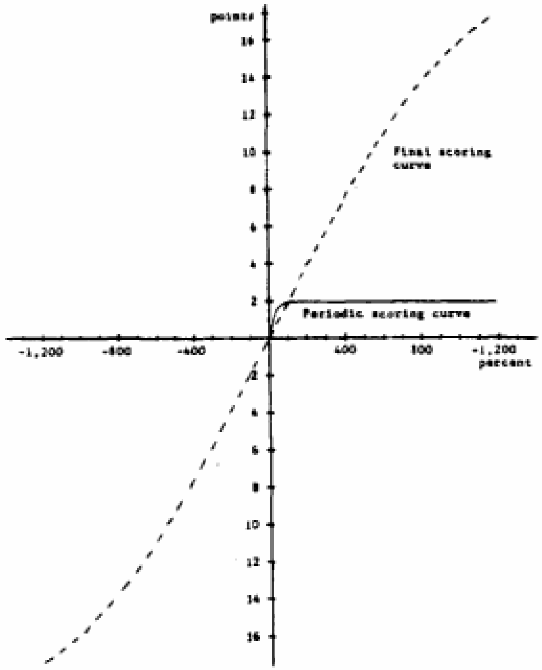
$$y = \frac{44}{1 + e^{-.1787x}} - 22. \tag{7}$$

FIGURE 10
DISTRIBUTION OF MONEY LEFT AT THE END



Note: Each X represents one player.

FIGURE 11
PERIODIC AND FINAL SCORING FUNCTION



CONCLUSION

We have shown how the logistic function has been used to score one computer-supported game that is genotypical to reality. Genotypical games, by their nature, bring to education a true reality that phenotypical games, i.e., simulations, cannot deliver. A significant problem in developing genotypical games has been the problem of scoring them. The use of the logistic function to solve this problem removes an impediment to the further development of such games.

The logistic function as we have defined it has characteristics desirable for grading purposes. It is simple, it is bounded by two horizontal asymptotes, it is symmetric about the origin, and its concavity above the ordinate tends to normalize a distribution of raw scores skewed towards the high end. Because of these characteristics, the logistic function is ideally suited to scoring business games on a periodic and cumulative basis.

REFERENCES

[1] Mincer, Jacob, "The Distribution of Labor Incomes: A Survey," Journal of Economic Literature, March 1970, pp. 1-26.

[2] Lekvall, Per and Clas Wahlbin, "A Study of Some Assumptions Underlying Innovation Diffusion Functions," Swedish Journal of Economics, Vol. 75, No. 4, 1973, pp. 362-377.

[3] Thavikulwat, Precha, "Boss: A Behavioral Quantitative, Computer-Supported Game, " in Lee A. Graf and David M. Currie (editors), Developments *in* Business Simulation & Experiential Exercises (The Proceedings of the Tenth Annual Conference of the Association for Business Simulation and Experiential Learning), Vol. 10, 1983, pp. 85-87.