DEMAND EQUATION REDUX: THE DESIGN AND FUNCTIONALITY OF THE GOLD/PRAY MODEL IN COMPUTERIZED BUSINESS SIMULATIONS

Steven C. Gold
Rochester Institute of Technology
sgold@saunders.rit.edu

Peter M. Markulis
SUNY Geneseo
markulis@geneseo.Edu

Daniel R. Strang
SUNY Geneseo
strang@geneseo.edu

ABSTRACT

This paper is a response to a paper entitled "Is the Gold/ Pray Simulation Model Valid and Is It Really Robust?" which was presented at the 2010 ABSEL conference by Kenneth Goosen. The Goosen paper called into question the validity and robustness of the Gold/Pray demand model. This paper is written as a response to the arguments advanced by Goosen. Goosen (2010) has raised five problems with the Gold/Pray (1983) model. The paper will proceed by listing and then replying to each of these alleged problems. One of the purposes of the paper is to encourage simulation developers, especially total enterprise simulation developers, to explain and discuss their underlying demand equations and how these equations have worked over the life of their simulation usage.

INTRODUCTION AND PURPOSE

In 1974, at the first annual ABSEL conference held in Oklahoma City, Churchill (1974) presented a small scale deterministic business game called DØG. In the paper he provided some of the details of the formulae and underlying demand functions that had been incorporated into the game. Since Churchill presented this information at the first annual ABSEL conference, it would not have been surprising to see a number of articles addressing demand functions over the course of ABSEL's thirty-seven year history. That has not been the case. Less than one percent of all the articles that have been published in the ABSEL's proceedings in the period of time 1974 to 2010 have focused on the demand functions that are the heart of business simulations. They are several plausible explanations for the paucity of articles on this subject. In 1981, Goosen alluded to the challenges in designing appropriate algorithms faced by potential simulation developers, and Pray and Gold (1982) state the challenges more explicitly and specifically. In his chapter devoted to designing business simulations, Teach (1990) indicates in spite of the fact that a great deal of information is provided about several of the simulations, the details of the demand algorithms *Markstrat* or *Industrat* have not been published. A perusal of the ABSEL literature draws one to the conclusion that simulation designers of business simulations have not been inclined to reveal the details of their demand functions in the literature; perhaps for proprietary reasons. Pray and Gold's use of the term black box is a result of the "secrecies of the internal workings" of simulations.

To focus the discussion of demand functions in simulations, it might be useful to state which, of the full range of simulations that fall in the purview of ABSEL, must have an integrated demand function in their underlying software. Biggs (1990) utilizes the definition that Cohen and Rhenmann (1961) presented when he describes a total enterprise game as one "designed to give people experience in making decisions at a top executive level and in which decisions from one functional area interact with those made in other areas of the firm." Keys and Biggs (1990) describe a total enterprise game as "one which includes decisions in most of the main functions of business: marketing, production, finance and personnel." Obviously simulations that are not total enterprise simulations might not have a built-in demand function, but equally obviously, all true total enterprise simulations have to address demand and have to have a demand function as an integral component. The following discussion of demand functions is limited to demand functions incorporated into total enterprise simulations.

In 1983, Gold and Pray presented their work on simulating market-level and firm-level demand. In their paper they indicate several of the alternative functional forms for demand models including a linear form, a non-linear form and a multiplicative form. In their article they discuss the advantages and disadvantages of each form and ultimately suggest the multiplicative form for several reasons, not the least of which is its robustness. Elements of robustness include the ability to eliminate conventional wisdom on the part of players which would severely limit the effectiveness of a simulation if it were discovered. In 1990, Gold and Pray discussed the potential problems of a simulation "blowing up!" Clearly, a robust demand model will prevent this potential unfortunate result for simulation play.

A perusal of the ABSEL literature indicates occasional interest in the modeling of demand functions in the period of time from 1974 to 2010. Each of articles provided were at best a marginal extension or reflection of the basic Gold/Pray demand algorithm. Decker, LaBarre and Adler (1987) presented two distinct approaches to defining the underlying functions of simulations, the multiplicative and the interpolation model. In 1998, Lambert and Lambert considered the advertising response in the Gold/Pray algorithm and Carvalho addressed the theoretical derivation of a basic demand function. Over the years, Teach (1984, 1986, 1990) has explored various facets of demand functions. In 2006, Murff, Teach, Schwartz present an algorithm they developed to establish industry-level demand in simulations. Murff et al. argue that their algorithm resolves the monotonicity problems attendant to the Gold/Pray algorithm which they maintain require artificial constraint of several of the keys decision variables. To that point, when the Gold/Pray demand algorithm is incorporated in a simulation, additional algorithms utilizing exponential smoothing of key variables can be embedded in the software to minimize or eliminate the adverse effects of extreme changes in the value of key variables. Although an exploration of these issues might be fascinating, it is not the purpose of this paper.

So, with the possible exception of the work of Murff, Teach and Schwartz, (2006) it is fair to say that ABSEL literature has added very little to understanding the modeling of the demand functions since Gold and Pray's work in 1983. In one respect, the Gold/Pray algorithm has been accepted as the standard for modeling demand in simulations.

In 2010, Goosen presented a paper at the annual ABSEL conference which called into question the validity and robustness of the Gold/Pray demand model. This paper is written as a response to the arguments advanced by Goosen. Goosen (2010) has raised five problems with the Gold/Pray (1983) model. The paper will proceed by listing and then replying to each of these alleged problems.

Impact of Exponential Smoothing on Marketing Expenditures
Table 1

Period	Marketing (Mn)	Exponentially Smoothed Marketing (M)		
		b = 0.4	b = 0.6	b = 0.8
1	$1,000	$1,000.00	$1,000.00	$1,000.00
2	$0	$700.00	$500.00	$200.00
3	$0	$490.00	$250.00	$40.00
4	$0	$343.00	$125.00	$8.00
5	$0	$240.10	$62.50	$1.60
6	$0	$168.07	$31.25	$0.32
7	$0	$117.65	$15.63	$0.06
8	$0	$82.35	$7.81	$0.01
9	$0	$57.65	$3.91	$0.003
10	$0	$40.35	$1.95	$0.001

FIRST PROBLEM: DOES THE GOLD/ PRAY MODEL ALLOW ADVERTISING OR R & D TO BE ZERO?

Goosen (2010) states "*The first problem concerns the effect of marketing (e.g., advertising) on demand. If advertising and R& D in the G/P model are zero, then demand is zero. There is no demand when price stands alone without advertising and R & D.*"

There are two reasons why this is not true. First the Gold/Pray model distinguishes between firm level demand and market level demand. With respect to firm level demand, the Gold/Pray model uses a weighting function which determines the firm's market share and its demand. In this weighting function there is a constant term added to each of the demand variables including price, advertising, and R & D. The equation is given as number 6 in Gold and Pray (1983) and is shown below as equation 1 of this paper:

$$(1)\ W_i = [P_i + k1]^{-(k2 + k3Pi)} [M_i + k4]^{+(k5 - k6Mi)} [R_i + k7]^{+(k8 - k9Ri)}$$

where:

W_i = weight of firm i
P_i = price of firm i
M_i = marketing expenditures of firm i
R_i = research and development expenditures of firm i
ki = parameters or constants i = 1 to 9

Referring to equation 1, the value for the firm's weight (Wi) determines the firm level demand. The constant terms associated with the demand variables of price (k1), marketing (k4), and research and development (k7) prevent the firm demand from going to zero, even if the demand variables are zero.

But what about the market level demand? Can the market level demand go to zero if all firms in the market charge a price of zero, and spend nothing on marketing and R & D? First, in the market demand equation "average"

Demand at three different levels of Marketing Expenditures
Table 2

	$50,000	**$75,000**	**$100,000**
Price	**Demand-50**	**Demand-75**	**Demand-100**
$180	1	3	5
$160	2	5	9
$140	3	9	17
$130	4	12	23
$120	5	16	30
$110	7	21	40
$100	9	28	54
$90	12	37	71
$80	16	48	93
$70	20	63	122
$60	26	82	158
$50	34	105	203
$40	43	134	259
$30	55	169	326
$20	68	209	404
$10	82	254	490

marketing expenditures and "average" R & D for the entire market is used. It is highly unlikely that all firms would decide not to advertise or do any R & D. Second, and more importantly, even if average values for marketing and R & D were zero in a period of the game, the market demand would not go to zero in the Gold/Pray model because the market level demand is a function of "exponentially smoothed" values for both market and firm price, marketing expenditures, and research and development expenditures. Equations 2, 3 and 4 in the Gold & Pray (1983) paper specify exponential smoothing for all demand variables and are shown below:

(2) $P = aP_n + (1-a)P_0$; where $0<a<1$
(3) $M = bM_n + (1-b)M_0$; where $0<b<1$
(4) $R = cR_n + (1-c)R_0$; where $0<c<1$

where:

P = exponentially smoothed price
M = exponentially smoothed marketing expenditures
R = exponentially smoothed R&D expenditures
a, b, c = smoothing coefficients
An "o" subscript indicates the last period smoothed value
An "n" subscript indicates the current period value

An exponentially smoothed value for P, M or R will take many periods to approach zero and will never have an exact value of zero.

As an example, let's look at the behavior of the exponentially smoothed marketing value (M) when marketing expenditures drop from $1,000 to zero (see Table 1) given three values for the coefficient "b" with respect to the exponentially smoothed marketing variable (M), i.e. 0.4, 0.6, 0.8. For example, in the case where the exponentially smoothed coefficient is 0.4 then the equation for the exponentially smoothed marketing variable becomes: $M = 0.4M_n + 0.6M_0$.

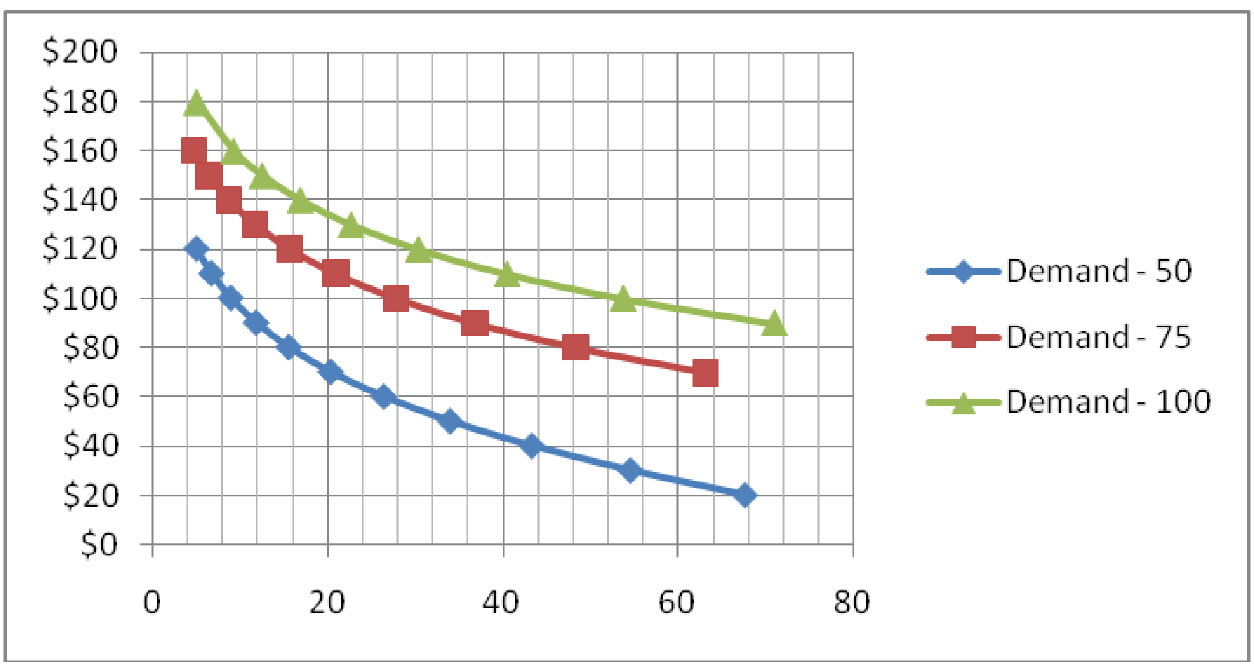
The exponentially smoothed value for marketing (M), which is used in the market demand model does not reach zero even after ten periods of zero expenditures. The lower the value of the coefficient for "b", the longer it takes for the exponentially smoothed value to approach, but never reach, zero.

Furthermore, the Gold/Pray (1983) market demand equation (number 3) in the model would allow a specification where each market variable for price, advertising, and development has a constant term associated with it, similar to the Gold/Pray firm demand equation 6.

SECOND PROBLEM: DOES THE GOLD/PRAY MODEL ALLOW FOR ONLY ONE KIND OF SHIFT IN THE DEMAND CURVE?

Goosen (2010) states "*The Gold/pray model does not allow a change in the Y-intercept. The Y-intercept is the price value on the Y axis where demand is zero.*"

Demand Shifts Caused by Increases in Marketing Expenditures
Figure 1



This is not correct. The Gold/Pray demand model does allow for a change in the Y-intercept. The market demand shifts upward/downward and to the right/left as the value for marketing expenditures (or research and development) change. To be perfectly accurate, the Gold/Pray demand function approaches the vertical price axis asymptotically, but never intersects the vertical axis. In this case, the price level where demand is close to zero, say 1 unit of demand, would be different depending on the level of the other non-price factors.

To illustrate, let's use the following Gold/Pray market demand function:

$$(4)\quad Q = g_1 P^{-(0.01 + 0.005P)} M^{+(3.9 - 0.0000015M)}$$

where $g1 = 1 \times 10^{-16}$

Using this equation and parameter values, we will solve for the relationship between price and quantity demanded at three different levels of marketing expenditures (M), i.e. $50,000, $75,000, and $100,000. Table 2 shows the demand relationship between price and quantity at these three different levels of marketing (M): (See Table 2 below)

The three different demand schedules (demand-50; demand-75; and demand-100) are graphed in Figure 1 to illustrate the impact of marketing expenditures. It is clear from the graph that the y-intercept increases with increases in marketing expenditures.

Based on the example equation, the price level required to restrict quantity demanded to approximately one unit is a price of $174 for demand-50, a price of $210 for demand-75, and a price of $230 for demand-100.

THIRD PROBLEM: DOES THE GOLD/ PRAY MODEL POTENTIALLY CREATE UNREALISTICALLY LARGE DEMAND?

Goosen (2010) states "*The Gold/Pray multiplicative model is driven by linear equations that serve as exponents. It should not be surprising then that a multiplicative model can have an explosive exponential effect. The multiplicative G/P model increases demand exponentially. Eventually, if advertising is increased enough the exponential increase will be staggering. Whether this problem of an explosive demand potential is an inherent flaw in the model or simply the result of a poor choice of parameters is at this point not clear*."

The explosive exponential effect of the Gold/Pray model illustrated by Goosen (2010) is simply a result of the choice of the selected parameters and not an inherent flaw in the model. To show explosive growth, Goosen uses the parameters derived from Table 3 below, which were taken from the example in the original Gold & Pray (1983) paper. In this example, the parameters were selected by the designer to have price elasticity that increases very rapidly from inelastic (-0.5) at a price of $10 to unit elastic (-1.0) at a price of $20. Also, the marketing and R & D elasticities were designed to start at very high levels of 3.0 at a price of $10.

To illustrate explosive growth, Goosen (1983) selected a price of $50, which according to the elasticity parameters in Table 3, would yield an extraordinarily high price elasticity; since price elasticity rises with the price level. It is unclear why a price of $50 was selected as the starting point, rather than a price between $10 and$20. But the important point is that with different parameter values,

Elasticity Parameters used in Goosen Example
Table 3

Price ($/unit)	Price Elasticity	Marketing ($)	Marketing Elasticity	R & D ($)	R & D Elasticity
$10	-0.5	$ 50,000	3.0	$ 50,000	3.0
$20	-1.0	$150,000	1.0	$150,000	1.0

Impact of significant increases in Marketing given constant price and R & D
Table 4

Marketing	R & D	Price	Demand (units)
$ 100	$100	$50	1,000,000
$ 200	$100	$50	1,331,969
$ 300	$100	$50	1,528,435
$1000	$100	$50	1,659,586

the demand function would be stable over a wide range of prices, and marketing and R & D expenditures.

As an alternative example, to illustrate the selection of more stable parameters, the following Gold/Pray demand function will be illustrated:

(5) $Q = g_1 P^{-(0.01 + 0.005P)} M^{+(0.5 - 0.0001M)} R^{+(0.5 - 0.0001R)}$; where $g_1 = 30320.2$

Following the same selection of pricing and marketing values presented by Goosen (2010) in his Table 2 we get the following results.

As evidenced in Table 4, the result of increasing marketing expenditures from $100 to up to $1,000 does not create an explosive result. Even doubling both marketing and R & D simultaneously does not create a problem of excess growth in demand as illustrated in Table 5.

Furthermore, the marketing expenditures would be exponentially smoothed before being placed in the demand equation, this would further dampen the impact increases in either marketing expenditures or R & D on firm demand. To illustrate assume the following exponential smoothing function for marketing expenditures:

(6) $M^e_t = bM_t + (1-b)M^e_{t-1}$

where:

M^e_t = exponentially smoothed value for M_t
M_t = marketing expenditures in period t
b = smoothing coefficient

Assuming a value of 0.5 for the smoothing coefficient (b), the impact of marketing expenditures on demand is shown in Table 5.

Using the exponentially smoothed value for marketing expenditures (M^e_t), and comparing Table 6 with Table 5, the impact on demand is significantly dampened. For example, increasing marketing expenditures from $1,000, only increases demand 1,749,278 units with exponential smoothing (Table 6) as compared to 2,754,228 units without exponential smoothing (Table 5).

FOURTH PROBLEM: DOES THE MODEL FOR BOTH MARKETING AND ADVERTISING CREATE BELL-SHAPED FUNCTIONS FOR MARKETING AND R & D?

Goosen (2010) states "*The G/P model actually creates a marketing function that is bell-shaped… there is still some doubt that the results are valid.*"

The quote from Goosen is arguing that the marketing literature does not support the possibility that excessive marketing expenditures could decrease a firm's demand. It is true that there is some controversy on whether excess advertising could cause firm demand to decline. As Goosen (2010) states the most accepted relationship is one of diminishing returns but not negative returns. Yet, there is evidence to the contrary. For example, a seminal paper by Dorfman and Steiner (1954) supports the possibility of negative returns to advertising.

Impact of significant increases in both Marketing and R & D given constant price
Table 5

Marketing	R & D	Price	Demand (units)
$ 100	$ 100	$50	1,000,000
$ 200	$ 200	$50	1,774,143
$ 300	$ 300	$50	2,336,113
$1000	$1000	$50	2,754,228

Impact of exponential smoothing on Marketing and Firm Demand
Table 6

Marketing (M_t)	Exponentially Smoothed M^e_t	R & D	Price	Demand (units)
$ 100	$100	$ 100	$50	1,000,000
$ 200	$150	$ 100	$50	1,189,609
$ 300	$225	$ 100	$50	1,390,487
$1000	$613	$100	$50	1,749,278

But the significant point with respect to the Gold/Pray demand model is that the parameters (coefficients) of the model can be set to eliminate negative returns to marketing or other variables. Referring to equation 1 above, the parameters k6 and k9 could be simply set to zero. This is demonstrated by re-writing equation 5 above as follows:

$$(7)\ Q = g_1 P^{-(0.01 + 0.005P)} M^{+(0.5 - 0.0M)} R^{+(0.5 - 0.0R)}$$

Note that the variable M in the exponent is multiplied by a zero parameter value, as well as the R variable. Equation 6 could then reduces to the following:

$$(8)\ Q = g_1 P^{-(0.01 + 0.005P)} M^{+(0.5)} R^{+(0.5)}$$

This form of the Gold/Pray function has diminishing returns to marketing and R & D but does not allow for negative returns at any level.

FIFTH PROBLEM: LACK OF PROOF THAT THE MULTIPLICATIVE MODEL IS DESIRABLE OR SUPERIOR

Goosen (2010) states "*The article by Gold and Pray in which their model was presented does not present any rationale as to why the multiplicative model is better. The article simply assumes that it is better.*"

This assertion by Goosen (2010) misrepresents the message given in the Gold & Pray (1983) article. The G/P paper never asserted their model was "superior". The G/P paper began by first outlining the advantages and disadvantages of alternative functional forms for modeling demand including: linear, non-linear, and multiplicative (page 102), and then suggesting a demand system that was argued to possess a number of desirable properties, but was never argued to be superior to all other demand models.

The major intent of the G/P paper was clearly stated in the conclusion as follows:

"*This paper represents an ongoing attempt to encourage open discussion concerning the design and development of computerized business simulations*". (p.106)

Proving that the Gold/Pray algorithm is desirable or even superior may truly not be achievable. However, in the 1983 article, Gold and Pray do present a number of compelling arguments for the merits of their algorithm.

Further, from an experiential or usage view, one can point to the many users of the DECIDE simulation by Pray and Strang (1980), published by McGraw-Hill, one of the premier academic publishing companies. DECIDE is a total enterprise simulation that has been used (and continues to be used) by several colleges and universities around the country (world), as well as in many consulting situations. The DECIDE simulation is based on a demand algorithm that utilizes most of the key elements of the Gold/Pray demand algorithm including a multiplicative demand function, demand functions that are generated at the industry level and the firm level, and exponential smoothing of the key input variables. McGraw-Hill required extensive alpha and beta testing with successful results as a prerequisite condition prior to commercialization of the software. DECIDE has now been used extensively over the course of thirty plus years in many business schools and according to the authors there has been **no** reports of the DECIDE simulation "blowing up" or yielding unrealistic results. Indeed, if DECIDE has worked without apparent flaws, it would follow that its underlying algorithms are sound. Although this is anecdotal evidence, its sheer volume supports the argument that the Gold/Pray demand algorithm is at least very sound, if not desirable and even, perhaps, superior.

CONCLUSION

The Goosen (2010) paper is consistent with the intent of the Gold & Pray (1983) article to encourage a healthy debate on the design of demand and other functions in computerized business simulations; and for this reason acknowledge the value of the contribution provided by Goosen.

We continue to encourage further discussion and debate (or critiques) of :

1. Our own demand equations;
2. Equations used by other simulation developers;
3. Results of long-term use of these equations in simulations.

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