RECIPES FOR STRUCTURAL FAIRNESS IN GAMES

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ABSTRACT

Although fairness is central to society and to games that are taken seriously, the structural aspect of fairness has not been addressed as a problem of games. Structural fairness in a game of multiple episodes with multiple parties contending for limited opportunities can be assured by an appropriate rotational procedure over a sufficient number of episodes. For fixed number of parties, positional rotation assures complete position fairness. In contrast, order rotation assures both complete positional fairness and complete order fairness, but only when number-of-party and number-of-episode conditions are satisfied. For variable number of parties, arrival rotation assures fairness to parties added last. Order rotation may assure fairness better than proportional allocation when opportunities cannot be distributed exactly in the proportions required. The Gold and Pray (1990) model can be adapted to include rotation. Structural fairness as equality of realized, rather than expected, opportunity cannot be assured by random selection; rotation is necessary.

INTRODUCTION

Consider a business game in which all firms require the same opportunities (or resources) from a single modeled entity. The firms are each managed by a team of students, who submit decisions for processing on an episode-by-episode basis, a classical business-game design pioneered in the early 1950s (Wolfe, 1993) and still in common use today. Suppose the opportunities required are labeled building permits and the modeled entity is said to be the government. In this case, the game would simulate the building construction industry. For this business game, and the many other games with a limited-opportunity element, the question of interest to game designers is how building permits (or opportunities) should be allocated to firms if the number of building permits is fewer than the total number sought by the firms.

The question of how limited opportunities should be allocated is the fundamental question in the study of economics, which generally prefers that the allocation should be based on free-market processes. In the case of building permits, the requirements of the firms might be addressed by asking the firms to bid for available permits. But if a free-market process is not built into the game, what allocation rule should be applied?

In fact, the use of free-market processes wherein players trade with other players (Cannon, Yaprak, & Mokra, 1999; Thavikulwat, 1997) is relatively new in business games. The vast majority of business games in common use apply mathematics to model the market (Cannon, Cannon, & Schwaiger, 2009; Cannon & Schwaiger, 2005; Gold & Pray, 2001; Goosen, 2009; Teach, 2007; Wolfe & Gold, 2007). Of the various models, the classic one is Gold and Pray's (1983, 1984, 1990), which models a market of sales opportunities in three-steps. The first step is to compute the number of opportunities (e.g., quantity demanded by the market) at the industry level (e.g., for all firms combined), the second step is to allocate the opportunities to the firms, and the third step is to reallocate opportunities in excess of product availability (e.g., stock outs) to the firms with a shortage from the second step, but only when stock outs are extreme. The three-step approach allocates opportunities without regard to product availability in the first two steps. It accounts for product availability in the third step, but only to the extent that opportunities greatly exceed product availability for some of the firms. Substituting the more general term *requirements* for *product availability*, the three-step approach may be said to be insensitive to requirements.

Requirements-insensitive allocation may be satisfactory for markets where opportunities are truly independent of requirements, so that fairness is not an issue. In the general case where opportunities depend upon requirements, however, fairness could be an issue. When fairness is an issue, fairness must rule, for as Rawls (1957, 2001) has argued, without fairness there can be no justice; without justice there can be no well-ordered society. Fairness may be especially important in a game, for although a game is an "activity standing quite consciously outside 'ordinary' life as being 'not serious'" (Huizinga, 1950, p. 13), games are supposed to be fair, so the players may have less tolerance for unfairness in a game than they would have in everyday-world activities.

WHAT IS FAIRNESS?

Fairness may be defined as equality of opportunity, a philosophical definition understood to represent a political ideal in a society of necessarily unequal opportunities (*Stanford Encyclopedia of Philosophy*, 2015). The definition begs the question of what constitutes an opportunity in a game of competing parties, where a party may be an individual, team, or community, depending upon the particulars of the game. If the opportunity is to be first—first to make a move, as in chess, or first to receive building permits, as in the supposed building-construction game—then only one party in a game can be first. If the toss of a coin should decide who is first, then that procedure may be acceptable to the players, but the fairness of the procedure depends on what happens later, for *when* things happen matters. In fact, fairness is about sequence, which together with synchronization and frequency constitutes the three elements of Moore's (1963) sociological analysis of time. If you hit me, that may not be fair. But if I hit you in return, then that should be fair.

More generally, fairness has two aspects: a structural aspect and a behavioral aspect. Structural fairness means that the architecture of the game is proper. Yet, even when the game is structurally fair, players may cheat, the behavioral aspect. This investigation, confined to structural fairness, recognizes but does not address cheating.

So, the problem of structural fairness may be rectified by rotating the sequence of play between the parties. In a game of two competing parties, rotation alternate the first-mover (or last -mover) advantage between the two parties, which requires the game to have an even number of episodes. In this case, the recipe is simple. If you make the first move in the first episode, I make the first move in the second episode, and so forth. For the building-construction game, if your building-permit requirements are satisfied first in the first episode, with permits remaining left for me; then my building-permit requirements should be satisfied first in the second episode, with permits remaining left for you.

But the recipe for a game of many competing parties is not so simple, for the number of ways many parties can be sequenced for rotation becomes rapidly very large as the number of parties rise, and only a subset of those ways are optimal. Thus, the number of ways by which a 6-party game can be sequenced is 6! = 720, too many to consider rotating through all of them in a single gaming event. The problem then is to identify the subset that is optimal for a realistic number of episodes in a single gaming event. As will be shown, the optimal subset for complete fairness in a 6-party game consists of only 6 episodes. More generally, any N-party game requires no more than $2N$ episodes for complete fairness.

KINDS OF STRUCTURAL FAIRNESS

Structural fairness is about sequence, the fairness of which depends upon what each party requires, what is available, and when each party arrives at the point of contention. In my view, a game of many parties and many episodes requires three kinds of structural fairness: positional fairness, order fairness, and arrival fairness.

Positional fairness is of concern when every party requires as much of the contested item as any other party. In this case, a party's position in the sequence is important but the identity of the party ahead is unimportant, because a difference in identity will not give rise to a difference in opportunity.

Order fairness is of concern when some party requires much more of the contested items than other parties. In this case, standing behind the high-requirements party is especially disadvantageous irrespective of one's position in the line.

Arrival fairness is of concern when different parties arrive at the distribution point at arbitrarily different times. In this case, giving early arrivals the advantage of earlier service would be capricious.

All three kinds of fairness can be resolved by rotation over a sufficiently large number of episodes. The objective is to maximize fairness over the fewest number of episodes.

The exposition that follows expands on each kind of fairness and explains how each is optimally addressed by a specific rotational procedure, or recipe. The exposition is technical, so those interested only in a conceptual understanding of how fairness might apply in a business game may skip the rest of this section and go directly to next section, Proportional Allocation, where the fairness-assuring methods are applied to the supposed building-construction game.

To minimize tedium, the exposition that follows assumes that the number of positions equals the number of parties. The assumption preserves generality, because dummy positions or dummy parties can be added for a perfect match whenever the number of positions do not equal the number of parties.

POSITIONAL FAIRNESS

I define complete positional fairness to mean that every party occupies each position in a sequence as frequently as any other party. The necessary condition for complete positional fairness is that the number of episodes must be an integer multiple of the number of parties. In a six-party, six-episode game, complete positional fairness is achieved by rotating the assignment of parties between episodes, as illustrated in Table 1, where the six parties are identified by the letters A through F. The rotation proceeds as follows:

1. Assign the parties by a convenient process, such as alphabetical order or drawing lots, to all the positions of the first episode.
2. Let $x_{i,j}$ refer to the party assigned to episode i and position j. Assign ordered letter to the parties of the first episode, A to $x_{1,1}$, B to $x_{1,2}$, C to $x_{1,3}$, and so on up to N, the number of parties, so the number of positions equals the number of parties.
3. Transpose the assignments of the first episode to the first position of N episodes, so the number of episodes also equals the number of parties.
4. Then for $i = 2$ through $i = N$ and $j = 2$ through $j = N$, assign $x_{i,j} = x_{i,j\text{-}1} + 1$, where $x_{i,j\text{-}1} + 1$ refers to the next letter after $x_{i,j\text{-}1}$, wrapping from the Nth letter back to the first letter when that next letter would exceed the Nth party.

TABLE 1
Complete Positional Fairness to Six Parties (A through F) Over Six Episodes

Episode	Position					
	1	2	3	4	5	6
1	A	B	C	D	E	F
2	B	C	D	E	F	A
3	C	D	E	F	A	B
4	D	E	F	A	B	C
5	E	F	A	B	C	D
6	F	A	B	C	D	E

This positional rotation procedure has an additive character that can been seen clearly if the parties are assigned integers, conveniently starting with zero (0), rather than letters such that 0 replaces A, 1 replaces B, 2 replaces C, and so on. Then the procedure reduces to Equation 1.

$$.x_{ij} = (i + j - 2) \bmod N \qquad (1)$$

The result is a Latin-square assignment that may be duplicated exactly in sets of Ns if additional episodes are desired. Equation 1 can be simplified by dropping the offsetting constant, in which case the formula becomes Equation 2. The effect of dropping the constant is to rotate all sequences by two positions, without losing positional fairness.

$$.x_{ij} = (i + j) \bmod N \qquad (2)$$

ORDER FAIRNESS

I define complete order fairness to mean that the relative place of a party to every other party occurs with the same frequency for all parties, where relative place refers to a party either preceding or following another party. In the case of the six-party, six-episode game arranged as shown in Table 1, A precedes B five time, in the first and third through sixth episodes, whereas B precedes A only once, in the second episode, so order fairness is incomplete.

In the case of the four-party, four-episode game arranged as shown in Table 2, order fairness is complete. To see this, note that the number of ordered pairs for any number (N) of parties is equal to $N \times (N - 1)$, so for 4 parties we have $4 \times 3 = 12$ ordered pairs, namely, AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, and DC. The AB order occurs in episodes 1 (A-B) and 3 (A-D-B), the AC order occurs in episodes 1 (A-B-C) and 2 (A-C), and so forth. Every ordered pair occurs twice. Similarly, order fairness is complete in the six-party, six-episode game arranged as shown in Table 3, for every ordered pair occurs thrice.

Positional fairness also is complete in both of these arrangements, as inspection verifies. To wit, consider any party, such as B. Notice that B appears once and only once in every position of both tables. The same is true for any other party, proving complete position fairness.

The arrangements of Tables 2 and 3 were generated by adjusting rotational assignments. The procedure proceeds as follows:

1. Take the first three steps of positional rotation.
2. Add a dummy $N + 1$ party.
3. Then for $i = 2$ through $i = N$ and $j = 2$ through $j = N$, assign $x_{i,j} = x_{i,j-1} + i$, where $x_{i,j-1} + i$ refers to the ith letter after $x_{i,j-1}$, wrapping from the $(N + 1)$th letter back to the first letter when that next letter would exceed the $(N + 1)$th party.

TABLE 2
Complete Positional and Order Fairness to Four Parties (A-D) Over Four Episodes

Episode	Position			
	1	2	3	4
1	A	B	C	D
2	B	D	A	C
3	C	A	D	B
4	D	C	B	A

TABLE 3
Complete Positional and Order Fairness to Six Parties (A-F) Over Six Episodes

Episode	Position					
	1	2	3	4	5	6
1	A	B	C	D	E	F
2	B	D	F	A	C	E
3	C	F	B	E	A	D
4	D	A	E	B	F	C
5	E	C	A	F	D	B
6	F	E	D	C	B	A

This order rotation procedure has a multiplicative character that can be seen clearly if the parties again are assigned integers rather than letters such that, as before, 0 replaces A, 1 replaces B, 2 replaces C, and so on. Then order rotation for $i < N + 1$ reduces to:

$$.x_{ij} = (ij - 1) \bmod (N + 1) \quad (3)$$

More generally, order rotation for any i is:

$$x_{ij} = (i'j - 1) \bmod (N + 1) \quad (4)$$

where

$$i' = [(i - 1) \bmod N] + 1 \quad (5)$$

Order rotation assures complete positional and order fairness under two conditions: (a) the numbers of episodes is an integer multiple of the number of parties and (b) $N + 1$ is a prime number. This assertion can be proven. To simplify the proof without losing generality, assume $i < N + 1$, enabling the proof to be based on Equation 3.

To prove positional fairness, consider Table 4, derived from $ij - 1$, Equation 3 before the modulus. Notice that the items in every row of Table 3 are transposed into every column, thus the items of row 3 (2, 5, 8…) are the same as the items of column 3 (2, 5, 8…). This is so because swapping i and j in $ij - 1$ leaves the results unchanged. Since every row is transposed into a column, proving that the N consecutive items of every row computed from Equation 3 are unique suffices to prove positional fairness.

TABLE 4
ORDER ROTATION BEFORE THE MODULUS

Episode	Position							
	1	2	3	4	5	6	7	8
1	0	1	2	3	4	5	6	7
2	1	3	5	7	9	11	13	15
3	2	5	8	11	14	17	20	23
4	3	7	11	15	19	23	27	31
5	4	9	14	19	24	29	34	39
6	5	11	17	23	29	35	41	47
7	6	13	20	27	34	41	48	55
8	7	15	23	31	39	47	55	63

If a row of Equation 3 should contain two items that are identical, then the difference between the dividends of both items must be $(ij_2 - 1) - (ij_1 - 1) = i(j_2 - j_1) = k(N + 1)$, where j_1 and j_2 refer to the positions of the two items and k can be any integer. The second equality is required for the modulus to yield the same item but the second equality is impossible when $N + 1$ is a prime number, which cannot be factored, because both i and j are less than $N + 1$.

Furthermore, the set of positive integers that can be residuals of any positive integer mod $(N + 1)$ is bounded by 0 and N, but the items ($x_{i,j}$) themselves are bounded by 0 and $N - 1$, so N must not fall within the bounds of $i < N + 1$ and $j < N + 1$. In fact, N lies just outside of the bounds, as the residual of the modulus $N + 1$ when $i = N + 1$ or $j = N + 1$ or both. Thus, the N consecutive items of every position of an episode must be unique and bounded by 0 and $N - 1$, proving positional fairness.

To prove order fairness, consider Table 5, derived from Equation 3 after applying modulus $N + 1 = 7$, thus $N = 6$. Notice that the items are inverted in positions 1 and 6, positions 2 and 5, and positions 3 and 4. Likewise, the items are inverted in episodes 1 and 6, episodes 2 and 5, and episodes 3 and 4. The inversions are specific to $N = 6$.

TABLE 5

Order Rotation After Modulo 7

Episode	Position							
	1	2	3	4	5	6	7	8
1	0	1	2	3	4	5	6	0
2	1	3	5	0	2	4	6	1
3	2	5	1	4	0	3	6	2
4	3	0	4	1	5	2	6	3
5	4	2	0	5	3	1	6	4
6	5	4	3	2	1	0	6	5
7	6	6	6	6	6	6	6	6
8	0	1	2	3	4	5	6	0

The inversions mean that the dividend of the inverted items differ by $k(N + 1)$, where k, as before, can be any integer. The inverse of episode i is episode $N + 1 - i = i''$ and the inverse of position j is $N + 1 - j = j''$, so the dividend-differences for inversion is as follows:

$$(i''j'' - 1) - (ij - 1) = (N + 1 - i) - (ij - 1) = k(N + 1) \qquad (6)$$

Equation 6 is true because its left side reduces to $(N - i - j)(N + 1)$, so every position truly is inverted in another position. This implies that whenever an item precedes another item in an episode, that item follows the other item in another episode, proving order fairness for any N items.

Accordingly, order rotation assures both complete positional fairness and complete order fairness when the number of episodes is an integer multiple of the number of parties and the number of parties is one less than any prime number, which covers Ns of 2, 4, 6, 10, and 12. By adding a step, both $N = 3$ and $N = 8$ also can be covered.

For $N = 3$, complete positional and order fairness can be achieved for episodes that are multiples of six by stacking two 3 × 3 Latin squares created by positional rotation such that the order of the parties of every episode is reversed between the two Latin squares. The result is a set of six sequences composed of all 3! = 6 possible sequences of 3 parties, so both positional and order fairness are assured.

Table 6 shows two stacked 3 × 3 Latin squares so constructed. The formula for the second $N \times N$ Latin square of the stack, $i >$ N, is given in Equation 7. As with Equations 1 and 2, Equation 7 also can be simplified by dropping the offsetting $N - 1$, in which case the formula becomes Equation 8.

TABLE 6

Stacked Arithmetic Rotation for Complete Positional and Order Fairness to Three Parties Over Six Episodes

Episode	Position		
	1	2	3
1	A	B	C
2	B	C	A
3	C	A	B
4	C	B	A
5	A	C	B
6	B	A	C

$$.x_{i,j} = (i - j + N - 1) \bmod N \qquad (7)$$

$$.x_{i,j} = (i - j) \bmod N \qquad (8)$$

The stacking procedure extends to Ns of any size. Stacking assures positional fairness because the first $N \times N$ Latin square is constructed by positional rotation and the second $N \times N$ Latin square merely inverts the order of the positions. Thus, in Table 6, the parties in the first position of the second Latin square are the parties in the third position of the first Latin square, and vice versa. Stacking also assures order fairness because each sequence of the second $N \times N$ Latin square is constructed by reversing the corresponding sequence of the first Latin square. So, complete positional fairness and complete order fairness is obtained in any N-party game in $2N$ episodes by positional rotation over the first N episodes and reversing the sequential ordering of the parties over the second N episodes.

For $N = 8$, eight is twice four, so an 8 × 8 Latin square can be constructed by tiling two 4 × 4 Latin squares on an alternating basis, each 4 × 4 Latin square constructed by including only four parties excluded from the other 4 × 4 Latin square. Table 7 shows the result of tiling that starts with the 4 × 4 Latin square of Table 2. So, for an eight-party game, tiling gives rise to complete positional fairness and complete order fairness in eight episodes, less than half the number of episodes that stacking requires.

Other cases generally require a choice between complete positional fairness and complete order fairness. Positional rotation assures complete positional fairness whenever the number of episodes is an integer multiple of the number of parties, because rotation causes each position to be occupied by the next party in the next episode. Order rotation by adding dummy parties, and positions, until the number of parties is one less than the next prime number assures complete order fairness when the number of episodes is an integer multiple of the number of parties, dummies included. Applying this last procedure to seven parties using Equation 3 and modulo 11 gives rise to Table 8, where the dummy parties appear as blanks in the table. Tightening the table by shifting parties to the left to occupy blanks, essentially skipping over dummies, gives rise to Table 9. Notice that the order of the parties in Table 9 is reversed between episodes 1 and 10, 2 and 9, 3 and 8, 4 and 7, and 5 and 6, so order fairness is complete.

TABLE 7
Tiled Order Rotation for Complete Positional and Order Fairness toEight Parties Over Eight Episodes

Episode	Position							
	1	2	3	4	5	6	7	8
1	A	B	C	D	E	F	G	H
2	B	D	A	C	F	H	E	G
3	C	A	D	B	G	E	H	F
4	D	C	B	A	H	G	F	E
5	E	F	G	H	A	B	C	D
6	F	H	E	G	B	D	A	C
7	G	E	H	F	C	A	D	B
8	H	G	F	E	D	C	B	A

TABLE 8
Order Rotation for 7 Parties over 10 Episodes Using Modulo 11 and Showing Dummy Parties as Blanks

Episode	Position									
	1	2	3	4	5	6	7	8	9	10
1	0	1	2	3	4	5	6			
2	1	3	5			0	2	4	6	
3	2	5		0	3	6		1	4	
4	3		0	4		1	5		2	6
5	4		3		2		1	6	0	5
6	5	0	6	1		2		3		4
7	6	2		5	1		4	0		3
8		4	1		6	3	0		5	2
9		6	4	2	0			5	3	1
10				6	5	4	3	2	1	0

ARRIVAL FAIRNESS

I define complete arrival fairness to mean that every party added to a sequence is inserted into a position with the same frequency as any other position in the sequence. For example, if party D is added to the XXX sequence, where each X represents a party of the existing sequence, D should appear in the first through fourth position with equal frequency. Thus, the sequence of four parties over four continuous episodes that yield complete arrival fairness could be DXXX, XDXX, XXDX, and XXXD, but it would not be DXXX, DXXX, XXDX, and XXXD. In the former case, party D appears once in the first through fourth position over the four episodes; in the latter case, party D appears twice in the first position and does not appear at all in the second position over the four episodes.

To construct the arrival-fair sequence for each episode, my formula, applied successively to each party from the first party (n = 1) to the last party (n = N) as that party is added to the sequence, is as given in Equation 9.

$$j = n - (i \bmod n) \quad (9)$$

TABLE 9
Order Rotation for 7 Parties over 10 Episodes Using Modulo 11 and Shifting Parties to Occupy Blanks

Episode	Position									
	1	2	3	4	5	6	7	8	9	10
1	0	1	2	3	4	5	6			
2	1	3	5	0	2	4	6			
3	2	5	0	3	6	1	4			
4	3	0	4	1	5	2	6			
5	4	3	2	1	6	0	5			
6	5	0	6	1	2	3	4			
7	6	2	5	1	4	0	3			
8	4	1	6	3	0	5	2			
9	6	4	2	0	5	3	1			
10	6	5	4	3	2	1	0			

For example, if the first episode (i = 1) involves three parties, the first party (n = 1) is assigned to position 1 – (1 mod 1) = 1 – 0 = 1. The second party (n = 2) is assigned to position 2 – (1 mod 2) = 2 – 1 = 1, which pushes the first party to the second position, so the two-party sequence is 10, or BA, where, as before, A = 0 and B = 1. The third party (n = 3) is assigned to position 3 – (1 mod 3) = 3 – 1 = 2, which pushes the first party from the second position to the third position, so the three-party sequence is 120, or BCA. Applying this procedure to six episodes gives rise to the set of sequences shown in Table 10.

TABLE 10
Arrival Rotation for Three Parties Over Six Episodes

Episode	Position		
	1	2	3
1	B	C	A
2	C	A	B
3	B	A	C
4	A	C	B
5	C	B	A
6	A	B	C

Arrival rotation applies particularly to multi-player games when players cycle through the game in waves, so that the first-arrival-first-served rule would be an unsuitable basis for adding players to a sequence constructed from the earlier wave, such as a multi-player game jointly played by classes that meet at different times. In this case, if first-arrival-first-served is used to sequence players for contested items, classes meeting earlier would be advantaged over classes meeting later, because the players of the earlier classes would, ceteris paribus, generally submit their decisions sooner and therefore be ahead in the queue than players of the later classes.

PROPORTIONAL ALLOCATION

Proportional allocation, wherein opportunities are allocated in proportion to requirements, may appear to be a simpler means of assuring fairness than rotation, but this is not so when opportunities cannot be allocated exactly in the proportions desired. To see why, consider again the building-construction game of the introduction. Suppose in every episode of the game, 10 building permits are available when the six firms of the game require a total of 21 building permits, distributed such that Firm A requires 1 permit; Firm B, 2 permits; Firm C, 3 permits; and so forth, as shown in Table 11. The objective is to fairly allocate the building permits to the firms over six episodes, given that the firms are the same in all other respects. This objective means that at the end of six episodes, the firms should have as many building permits in proportion to the total the number of building permits issued, as the firms require in proportion to the sum of the requirements of all firms. Thus, the proportion of permits distributed across the six firms after six episodes should be as close as possible to the proportion of requirements shown in Table 11.

Table 12 shows the distribution of permits resulting from positional rotation, applying the sequences of Table 1. In this case, for episode 1, the distribution begins with Firm A, which gets 1 permit; followed by Firm B, 2 permits; Firm C, 3 permits; and Firm D, 4 permits. The 10 available permits having been distributed, no permit remains for Firm E and Firm F. In episode 2, the distribution begins with Firm B, which gets 2 permits; followed by Firm C, 4 permits; and so forth as shown in Table 12.

Table 13 shows the distribution of permits resulting from order rotation, applying the sequences of Table 3. Table 14 shows the distribution of permits resulting from arrival rotation, extending the sequences of Table 10 from three parties over three episodes to six parties over six episodes. After 6 episodes, 60 permits will have been distributed in the proportions shown in Tables 12 through 14 for the three rotational methods.

As to proportional allocation, allocating the 10 building permits in proportion to requirements means that the permits should be allocated as shown in the last two rows of Table 11, depending on the divisibility of permits. Considering that building permits should be indivisible, the allocation could be adjusted to give Firm A one permit instead of zero, so that the sum of all permits proportionally allocated equals the 10 available. The adjustment resolves a problem of proportional allocation: When the opportunities are in indivisible units, the sum of opportunities allocated may not equal the number of opportunities available. The result of applying adjusted

TABLE 11
Distribution of Permits Required and Available

	A	B	C	D	E	F	Sum
Permits required	1	2	3	4	5	6	21
Proportion of required	0.048	0.095	0.143	0.190	0.238	0.286	1.000
Permits available, raw	0.476	0.952	1.429	1.905	2.381	2.857	10
Permits available, rounded	0	1	1	2	2	3	9

TABLE 12
Distribution of Permits by Positional Rotation

Episode	Sequence	A	B	C	D	E	F	Sum
1	A-B-C-D-E-F	1	2	3	4	0	0	10
2	B-C-D-E-F-A	0	2	3	4	1	0	10
3	C-D-E-F-A-B	0	0	3	4	3	0	10
4	D-E-F-A-B-C	0	0	0	4	5	1	10
5	E-F-A-B-C-D	0	0	0	0	5	5	10
6	F-A-B-C-D-E	1	2	1	0	0	6	10
	Sum	2	6	10	16	14	12	60
	Proportion of Total	0.033	0.100	0.167	0.267	0.233	0.200	1.000

TABLE 13
Distribution of Permits by Order Rotation

Episode	Sequence	A	B	C	D	E	F	Sum
1	A-B-C-D-E-F	1	2	3	4	0	0	10
2	B-D-F-A-C-E	0	2	0	4	0	4	10
3	C-F-B-E-A-D	0	1	3	0	0	6	10
4	D-A-E-B-F-C	1	0	0	4	5	0	10
5	E-C-A-F-D-B	1	0	3	0	5	1	10
6	F-E-D-C-B-A	0	0	0	0	4	6	10
	Sum	3	5	9	12	14	17	60
	Proportion of Total	0.050	0.083	0.150	0.200	0.233	0.283	1.000

proportional allocation to six episodes is shown in Table 15.

The root-mean-squared difference between the proportion of permits required (Table 11) and the proportion of total permits distributed over six episodes applying positional rotation (Table 12), order rotation (Table 13), arrival rotation (Table 14), and adjusted proportional allocation (Table 15) is shown in Table 16. Order rotation gives rise to a difference of .007 that is less than a quarter of the difference of the next smallest difference of .033, the result of adjusted proportional allocation. A graph of the differences is shown in Figure 1. Clearly, order rotation is the method that most closely matches requirements, so it is the fairest of the four allocation methods for the building-construction game. The superior performance of order rotation over proportional allocation does not extend to all cases of indivisible limited opportunities, but the fact that order rotation can sometimes be better should deter game designer from uncritically relying on proportional allocation.

TABLE 16
Root-Mean-Squared Difference of Allocation Methods

Method	Difference
Positional rotation	.048
Order rotation	.007
Arrival rotation	.052
Proportional allocation	.033

ADAPTING THE GOLD AND PRAY MODEL

Returning to the Gold and Pray (1983, 1984, 1990) model, the model applies proportional allocation as its second step and is based on a log-linear demand function that assumes that demand is independent of supply, which is to say, more generally, that opportunities are independent of requirements. The supply-independence assumption may be less generally valid than it appears, considering the common observation that street-fair stalls displaying more items of the same product sell more items than stalls displaying fewer items. The greater demand for the products of the stalls with more items may be attributed to greater visibility of the items displayed and more confidence among shoppers that the product is worth the price, because the vendor has evidently committed more resources to the product.

So, if opportunities should depend upon requirements, the question of interest to game designers is how Gold and Pray's (1983, 1984, 1990) requirements-insensitive approach might be modified to include requirements more directly within the model, ergo, to allow demand to depend on supply. One way to do this is to add supply as an independent variable of the demand function. In this case, Gold and Pray's (1990, p. 123) log-linear demand function becomes Equation 10, where Q is the quantity demanded; P, price; M, marketing expenditure; R, research and development expenditure; S, supply; and a, b, c, e, and g are parameters. This method treats supply as simply another independent variable, without recognizing supply's unique role in sales.

$$Q = aP^{-b}M^{c}R^{e}S^{g} \qquad (10)$$

Another way is to keep the original demand function, but to substitute order rotation for the third step, when stock outs are reallocated. Thus, the sum of quantities demanded in excess of quantities available would be distributed by order rotation to firms wherever quantities available exceed quantities demanded, forgoing Gold and Pray's (1990, p. 135) checks for extreme values. The modified model can be tuned to put more weight on supply by including only a fraction of industry-level demand in the second step, when industry-level demand is allocated to firms, leaving the balance for distribution by order

TABLE 14
Distribution of Permits by Arrival Rotation

Episode	Sequence	A	B	C	D	E	F	Sum
1	B-C-D-E-F-A	0	2	3	4	1	0	10
2	C-D-E-F-A-B	0	0	3	4	3	0	10
3	D-E-F-B-A-C	0	0	0	4	5	1	10
4	E-F-A-C-B-D	0	0	0	0	5	5	10
5	F-C-B-D-A-E	0	1	3	0	0	6	10
6	A-D-B-E-C-F	1	2	0	4	3	0	10
	Sum	1	5	9	16	17	12	60
	Proportion of Total	0.017	0.083	0.150	0.267	0.283	0.200	1.000

TABLE 15
Distribution of Permits by Adjusted Proportional Allocation

Episode	A	B	C	D	E	F	Sum
1	1	1	1	2	2	3	10
2	1	1	1	2	2	3	10
3	1	1	1	2	2	3	10
4	1	1	1	2	2	3	10
5	1	1	1	2	2	3	10
6	1	1	1	2	2	3	10
Sum	6	6	6	12	12	18	60
Proportion of Total	0.100	0.100	0.100	0.200	0.200	0.300	1.000

rotation. This second method ties demand to stock outs, so it is consistent with the view that customers preferentially shop at firms with high supply because high-supply firms are less likely to have stock outs.

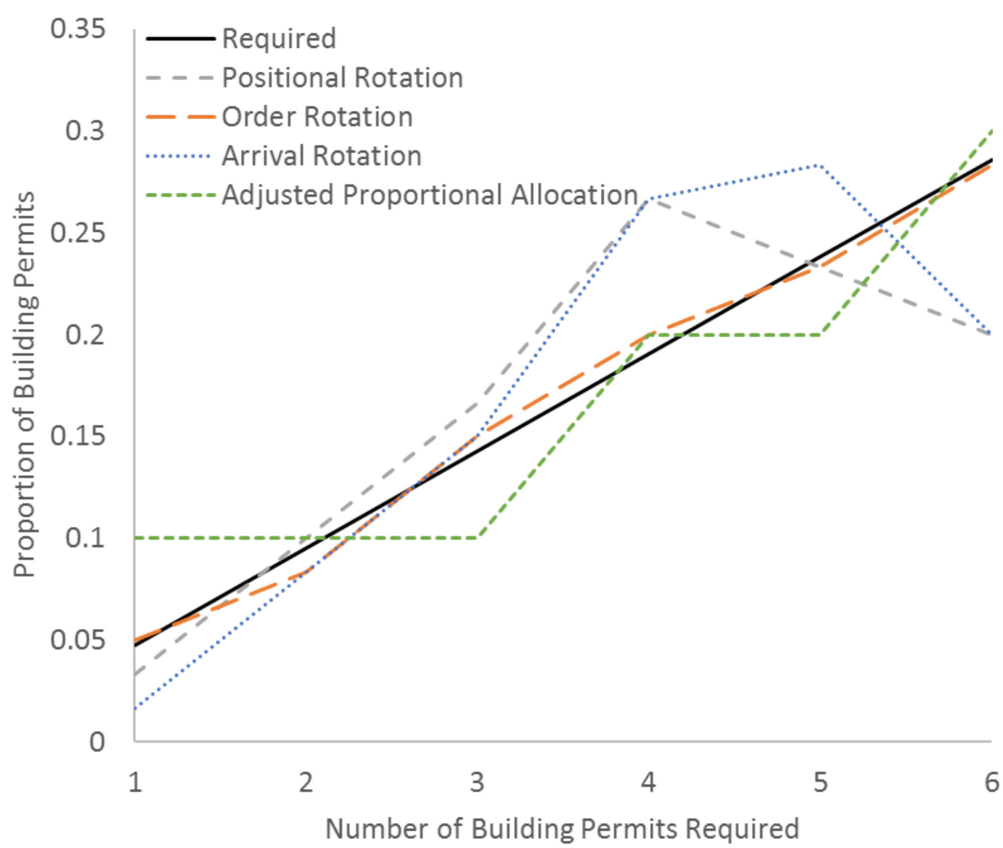
CONCLUSION

Fairness, as equality of *realized* opportunity, cannot be achieved by random selection, for random selection equalizes *expected* opportunity. True fairness requires that opportunities be realized, not merely expected. The way to true fairness is through recipes that assure that realized opportunities are as equal as they can be within the constraints of any particular game. Positional rotation, order rotation, and arrival rotation are three new recipes for fairness in games of more than a single episode. Order rotation is the fairest rotational method of all when two conditions are satisfied. First, the number of episodes should be an integer multiple of the number of parties. Second, the number of parties should be 3, 8, or one less than a prime number.

The number of episodes in a game is generally set by the game's administrator rather than fixed by its designer, but the design of the game limits what is possible. Typically, business games are administered for 4 to 12 episodes (Anderson & Lawton, 1992; Rollier, 1992). One might argue that if a game is administered for many more episodes, the law of large numbers will assure a fair outcome with random allocation, which would be simpler to administer than rotation. The argument is fallacious, because large numbers do not eliminate runs, which is why, as Styer (2000) has observed, stars are not uniformly distributed in the sky even though stars are born of a random process. Moreover, the relative performance of competing parties tends to be correlated across the episodes of business games, which is why early dominance has been shown to be a notable problem (Bernard & de Souza, 2009; Patz, 1992, 1999, 2000; Peach and Platt, 2000; Rollier, 1992; Teach & Patel, 2007), even if the problem is not always found (Wolfe, Biggs, & Gold, 2013). Random selection gives rise to runs that accentuate any advantage, and proportional distribution fixes any advantage in place. Rotation forestalls runs, ameliorating advantage.

Further research on fairness in games might proceed towards clarifying the conditions under which each method yields the smallest root-mean-squared difference between the proportion of opportunities required and the proportion of opportunities distributed. For now, the conclusion is that games can be structurally fair, but the game that is structurally fair must be a multi-episodic game that incorporates fairness into its design.

FIGURE 1
Proportional Differences of Allocation Methods by Required Building Permits



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