THE GOLD/PRAY DEMAND MODEL REVISITED: A RESPONSE TO A RESPONSE

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ABSTRACT

In 2010, I presented a paper at ABSEL in which I identified and discussed some problems which potentially reside in the Gold/Pray demand model. The next year at ABSEL, Gold, Markulis and Strang presented a paper in which they responded to the issues I raised. They concluded that the issues raised could be eliminated by selection of different parameters or that I had incorrectly understood their demand algorithm. In one instance, their solution created a new problem. Their paper was a good response and gave a better understanding and new insights into the Gold/Pray demand algorithm. In one sense, their paper was an addendum to the Gold/Pray 1983 paper**.** The purpose of this paper is to show that some of the problems I identified still remain in spite of the GMS authors belief that they had satisfactorily dismissed the issues I originally discussed.

INTRODUCTION

In 2010, I presented a paper at ABSEL in which I identified and discussed some problems which potentially reside in the Gold/Pray demand model. The next year at ABSEL, Gold, Markulis and Strang presented a paper in which they responded to the issues I raised. They concluded that the issues raised could be eliminated by selection of different parameters or that I had incorrectly understood their demand algorithm. In one instance, their solution created a new problem. Their paper was a good response and gave a better understanding and new insights into the Gold/Pray demand algorithm. In one sense, their paper was an addendum to the Gold/Pray 1983 paper**.** The purpose of this paper is to show that some of the problems I identified still remain in spite of the GMS authors belief that they had satisfactorily dismissed the issues I originally discussed.

BACKGROUND

Prior to 1983, there had been no significant discussion of how the authors of business simulations developed market demand algorithms. Simulation authors were reluctant to reveal the industry and firm demand algorithms they used or created. In one sense, the mathematical models were kept secret in a black box. Users basically only knew what inputs were required and what output was generated. Few if any papers ever questioned the validity of demand models or discussed output problems created by the existing demand models. This ended in 1982 when Steven Gold and Tom Pray presented a paper which analyzed the demand algorithms of eight existing business simulations. Then in 1983 the same two authors presented a best award paper in which they introduced a landmark multiplicative demand algorithm. However, the extent to which their model may have been used later by simulation authors in not known. In my search using the BKL library, I found over one hundred papers that made a reference to the Gold/Pray demand model; however, I was unable to find any papers that reported incorporating the model into a new simulation. The Gold/Pray algorithm involves some complex mathematics and for a reader to understand the paper, some proficiency in algebra fundamentals is required. The demand equations require an understanding of exponents and the ability to solve simultaneous equations. In addition, a solid understanding of the basic fundamentals of price theory is required. Nevertheless, the 1983 paper provided a foundation for additional papers on market demand algorithms. Following 1983, several modifications or alternative demand algorithms were presented in papers at ABSEL (Decker, 1987) Carvalaho,1991; Dickinson, 2014, Goosen, 2010; Teach (2000).

The Gold/Pray demand algorithm consisted of three basic equations. One equation for market demand and two equations for firm demand:

Market Demand Equation

$$Q = g1P^{-(g2 + g3P)}M^{+(g4 - g5M)}R^{+(g6 - g7)}$$

Firm Demand Equation

$$W_i = K_o(P_i + K_i)^{-k_2 + K_3 Pi)}(M_i + K_4)^{+(K_5 + K_6 Mi)}R_i + K_7)^{+(K_8 + K_9 Ri)}$$

Market Share Equation

$$S_i = W_i / \sum_{I=1}^{N} W_i$$

The Gold/Pray algorithm consisted of a series of equation that created market demand and then allocated that demand to the firms in the simulation**.** How the parameters were determined involved a mathematical process of setting elasticity values and then solving simultaneous elasticity equations. Gold and Pray described their demand algorithm as one which was "a generalized multiplicative market demand function which allows for variable elasticity and a multiplicative firm level demand function which has variable firm level elasticity and is constrained by the total market demand."

In my 2010 paper, I identified five problems that needed analysis and solutions. Gold, Markulis, and Strang wrote a response paper in 2011 which they believed that had demonstrated that these problems did not exist or could easily be eliminated. Following is my analysis of their response paper. In the discussion to follow, I will refer to Gold, Strang, and Markulis as the GMS authors. Also, the GMS authors use the term "Marketing" as a demand variable. My use of the term "Advertising" means the same as the GMS use of the term "Marketing". The term marketing is much broader than advertising and includes such decisions as price, sales force, sales people salaries, and research and development. Price, advertising, and R & D are independent decisions in most simulations. This paper is primarily concerning with the effect of advertising and R &D decisions on demand. Given the analysis involved in this paper, the use of the term advertising is more appropriate than the much broader term marketing.

FIRST PROBLEM: DOES THE GOLD/PRAY MODEL ALLOW ADVERTISING OR R & D TO BE ZERO?

> Goosen (2010) states "*The first problem concerns the effect of marketing (e.g., advertising) on demand. If advertising and R& D in the G/P model are zero, then demand is zero. There is no demand when price stands alone without advertising and R & D.*"

The authors have skirted around my statement of this problem and have introduced discussion and examples that have nothing to do with my alleged problem. The first problem concerned the effect on industry demand when advertising and R & D are zero and not that they might be zero. The authors presented two reasons why my analysis was incorrect concerning zero values. First, the reason given had to do with firm level demand. In essence the authors say that the firm level equation includes a constant term that prevents zero industry demand. This may be true for firm demand but my problem with the Pray/Good demand algorithm had nothing to do with firm level demand. Rather my concern was only with the industry demand equation.

Concerning my problem with the effect of advertising and R & D being zero on industry demand, the authors stated, "It is highly unlikely that all firms would decide to not advertise or do any R & D." My argument concerned advertising and R & D in fact being zero and did not concern them being unlikely. My point was that if advertising and R & D were zero, then price alone would

TABLE 1
GOLD/PRAY PRICE DEMAND CURVE

g1 = 30320.2	g2 - .01	g3 = .005	g4 = .5	g5 = .0001	g6 =.5 g7 = .0001
Price	Advertising	R & D	Quantity	Dmd. Increase	Revenue
100	1	1	2,896		$289,500
90	1	1	3,826	930	$344,340
80	1	1	5,028	1,202	$402,340
70	1	1	6,568	1,540	$459,760
60	1	1	8,521	1,953	$511,260
50	1	1	10,964	2,443	$548,200
40	1	1	13,973	3,009	$558,920
30	1	1	17,795	3,822	$533,850
20	1	1	21,808	4,013	$436,160
10	1	1	26,407	4,599	$264,070
1	1	1	30,320	3,913	$30,320

not generate any demand based on the demand parameters used in the original paper. The author also said:

> *Second and more importantly, even if average values for advertising and R & D in a period of a game were zero, the market demand would not go to zero in the Gold/Pray model because the market level demand is a function of exponentially smoothed values for both market and firm price, marketing expenditures, and research and development expenditures.*

This explanation would not be true if in the first period either average advertising or average R & D were zero, but would only be true if in period 1 advertising and R & D were positive. Regardless of exponential smoothing, in the first period with advertising or R & D being zero, industry demand would be zero in period 1 given the original parameters in the 1983 Gold/Pray paper.

One of the arguments I made was that advertising and R & D could never be zero. Obviously, this is true. Assume the equation Q = P x A x D. Now assume that P is 10 and A is 5 while D is 0. Then Q = 10 x 5 x 0 = 0. Regardless of any value assigned to the demand parameters, if either average advertising or average R & D is zero, then demand is zero. The GMS authors are correct in saying that zero values in actual simulation play are unlikely. However, zero average advertising or R & D are still a theoretical possibility.

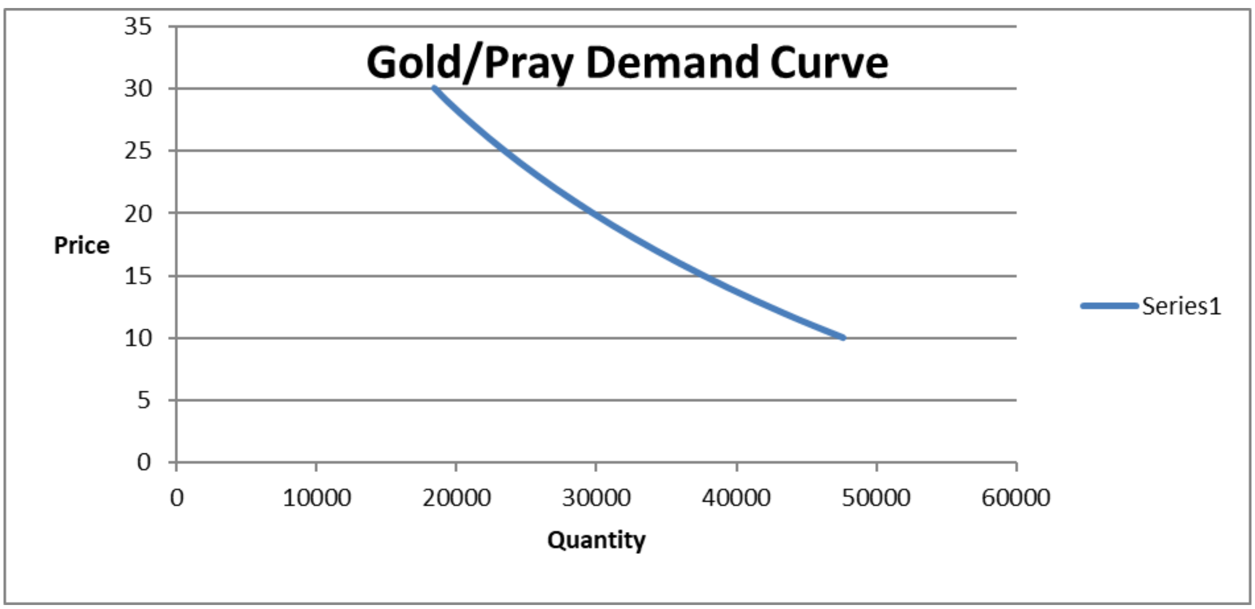
One of my goals in my 2010 paper was to determine the price demand curve independent of advertising and R & D. I realized that I could not derive the price demand curve if I let either advertising or R & D be zero. What I failed to realize was that if I set both advertising and demand to 1, then I could determine the Gold/Pray demand curve. The GMS authors in their discussion of my problem three, set forth a revised industry demand equation:

$$Q =- g1P-(.01 + .005P)M+(.5 - .005M)R+(.5-.0001R)$$

Using this equation, I was able to derive the G/P demand curve if we let advertising =1 and R & D = 1. The results of letting price vary from $100 to $10 are shown table 1.

The G/P demand curve can be shown as a chart as follow:

FIGURE 1



Now this demand curve was based on advertising and R & D being equal to 1. What if advertising and R & D both were given a value of $100. At a price of $20, industry demand becomes 1,988,926. The increase in demand from 4,013 now to demand of 1,988,925 is rather dramatic.

SECOND PROBLEM: DOES THE GOLD/PRAY MODEL ALLOW FOR ONLY ONE KIND OF SHIFT IN THE DEMAND CURVE?

> Goosen (2010) states "*The Gold/pray model does not allow a change in the Y-intercept. The Y-intercept is the price value on the Y axis where demand is zero.*"

This problem I raised is very subtle and requires some background knowledge to understand. If the demand curve is linear as typically presented in economic textbooks, it is beyond dispute that the Y-intercept does not change if advertising causes a constant percentage change shift in the demand curve. If the Y-intercept is $110 before advertising, then it is still $110 after advertising. However, the Gold/Pray industry demand equation is non-linear in nature and the authors are correct in saying the Gold/ Pray demand curve is asymptotically and never quite intersects the vertical axis. Nevertheless the linear demand curve and the Gold/ pray demand curve have one thing in common. The shift in the demand curves is a constant percentage shift. When a shift results from advertising in the linear demand curve, the change is a constant percentage. If at a price of $80 demand increases by 20%, then at a price of $100 or $60, for example, the shift is still a 20% shift. If demand at $100 is 1,000, and demand at a price of $60 is 3,000, then demand after the shift would be 1,200 at price of $100 and 3,600 at a price of $60. In both cases, the increase in demand is 20%.

Now the same is true of the Gold/Pray model. In their Table 2 example, the GMS authors show that at advertising of $50,000 and a price of $110 industry demand would be 7. Then at advertising of $75,000 and also price of $110 demand would 21. The resulting increase in demand was 200% (14/7). Now at a price of $80, before advertising of $50,000, demand was16. After advertising of $75,000, demand at a price of $80 was 48. Consequently, again the percentage increase in demand was 200% (32/16). In fact, after advertising of $75,000 , the increase in demand is always 200% at all prices.

Technically, the authors are correct that after an increase in advertising the Y-intercept does not remain the same. In one sense, there is never a Y-intercept in the Gold/Pray model. A more enlightening point I should have made was to say that in the Gold/Pray model a shift caused by an increase in advertising creates a constant percentage shift at each and every price after a given increase in advertising. In this respect, linear demand curves and the Gold/Pray demand curve are the same. At prices substantially higher prices than the GMS $180 price that resulted in a demand of 1 unit, the change is demand is very small. Perhaps it would be essentially correct to say that a small change in demand is the equivalent of an intersection with the vertical axis has happened. In the GMS authors Table 2, demand at different amounts of advertising gave the following demand results when price was $180:

Advertising	Demand
$50,000	1
$75,000	3
$100,000	5

The author say that this is proof the Y-intercept is increasing. This statement is misleading. At a price of $200 and the same advertising amounts as above, these results occur:

Advertising	Demand
$50,000	0.45
$75,000	1.38
$100,000	2.69

Actually, it would be more correct to say that as price is increasing the non-existent "Y-intercept " is decreasing.

THIRD PROBLEM: DOES THE GOLD/PRAY MODEL POTENTIALLY CREATE UNREALISTICALLY LARGE DEMAND?

Goosen (2010) states "*The Gold/Pray multiplicative model is driven by linear equations that serve as exponents. It should not be surprising then that a multiplicative model can have an explosive exponential effect. The multiplicative G/P model increases demand exponentially. Eventually, if advertising is increased enough the exponential increase will be staggering. Whether this problem of an explosive demand potential is an inherent flaw in the model or simply the result of a poor choice of parameters is at this point not clear*."

Regarding the third problem Gold, Strang, Markulis state:

"The explosive exponential effect of the Gold/Pray model illustrated by Goosen (2010) is simply a result of the choice of the selected parameters and not an inherent flaw in the model."

On this point we are in total agreement. The Gold/Pray 1983 article would have been better if different parameters had been selected. The authors then further state:

"But the important point is that with different parameter values, the demand function would be stable over a wide range of prices, and marketing and R & D expenditures."

The authors then use a different set of parameters to demonstrate that their market demand equation would generate "stable" results over a wide range of price, advertising and, R & D decisions. The authors then presented in their Table 4 (my Table 2) which they say confirms their contention of stability. In my opinion, their Table 4 actually disproves their contention of stability and confirms my point that their model inherently has the potential to create unreasonable demand values. The original demand parameters and the new selected parameters were as follows:

	g1	g2	g3	g4	g5	g6	g7
Original	.00000000000234	.15	.01	3.88	.0000015		
New	30320.2	.01	.005	.5	.0001	.5	.0001

The revised equation consequently becomes: $Q = 30320.2P^{-(.01 + .005P)}M^{+(.5 - .005M)}R^{+(.5 - .0001R}$). The numbers the authors presented in their Table 4 (my Table 2) appear on the surface to be reasonable demand responses at the price, advertising, and R & D amounts assumed. At price of $50, advertising of $100, and R & D of $100, market demand is shown as 1,000,000.

TABLE 2
(TABLE 4 FROM PAPER BY GOLD, MARKULIS, STRANG)

Marketing	R & D	Price	Demand
$100	$100	$50	1,000,000
$200	$100	$50	1,331,96
$300	$100	$50	1,528,425
$1,000	$100	$50	1,659,586

To achieve these results in was necessary to change g1 from .00000000000234 to 30320.2 and also change starting advertising from $50,000 in their original paper to $100

The problem is that the new demand of 1,000,000 at a price of $50 and both advertising and R & D being equal to $100 is not reasonable. Given the price of $50 and at industry demand of 1,000,000, then total industry sales would be $50,000,000. Now in an industry that has total sales of $50,000,000, it is not reasonable to believe that total advertising and R & D of just $200 could generate such a large amount of sales and create an explosively large demand of 1,000,000 units. Statistics concerning advertising as a percentage of sales reveal that advertising typically varies from 7% to 10% of sales in most businesses. Assuming 7% to be reasonable, then it should take average industry advertising of $3,500,000 of industry advertising to generate industry demand of 1,000,000 units or sales of $50,000,000. In my judgment, the GMS authors' Table 4 failed to refute my argument that the Gold/Pray demand equation by proper selection of parameters can avoid explosive industry demand values.

Now in their Table 4, (my Table 2) the authors kept price constant at $50 and R & D constant at $100. Marketing (advertising) ranged from $100 to $1,000. Keeping one decision variable constant will automatically reduce the explosive effect on market demand. What if R & D is also made equal to advertising and we let advertising and R & D increase from $1,000 to $10,000? We then will get the following results:

TABLE 3
RESULTS: GMS NEW PARAMETERS G1 = 30320.2 G2 = .01 G3 =.005 G4 = .5 G5=.0001 G6 =.5 G7 = .0001

	Price	Advertising	R & D	Demand	Revenue
	1	2	3	4	5
1	$50	$1	$1	10,965	$548,250
1	$50	$100	$100	1,000000	$50,000,000
2	$50	$200	$200	1,774,143	$88,707,150
3	$50	#300	$300	2,336,113	$116,805,650
4	$50	$400	$400	2,715,783	$135,789,150
5	$50	$500	$500	2,944,916	$147,245,00
6	$50	$600	$600	3,053,324	$152,666,200
7	$50	$700	$700	3067497	$153,374,850
8	$50	$800	$800	3,010,201	$150,510,050
9	$50	$900	$900	2,900,539	$145,026,950
10	$50	$1,000	$1,000	2,754,200	$127,710,000
	$50	$2,000	$2,000	1,048,619	
11	$50	$10,000	$10,000	1	$50

In my judgment, the results shown in Table 3 confirm the the potentially explosive nature of the Gold/Pray demand equations even given the new parameters. The new parameter values failed to prove that the potential explosive nature of the Gold/ Pray demand equation can be controlled. A close examination of the above table reveals even more surprising results. When advertising and R & D are a total of $2.00 industry demand is 10,965. If advertising and R & D are increased to a total of $200, then demand becomes 1,000,000 units and total sales become $50,000,000. It would appear that an increase in sales from $548,250 to $50,000,000 by advertising and R & D of $200 can be only described as rather explosive.

A second unexpected demand occurrence happens when advertising and R & D are increased to $10,000. At advertising of $10,000 and R & D of $10,000 demand is only 1 unit. That at some level of advertising and R & D (for example $10,000) can cause demand to be almost zero is a result that is surely is undesirable. Table 3 shows a degree of instability in the Gold /Pray industry demand equation contrary to one of the stated benefits of the assignment of different parameters to the original Gold/Pray equation.

When advertising is increased from $1,000 to $2,000 and also R & D from $1,000 to $2,000 there is a 1,705,608 decrease in industry demand , a more than a 50% decrease. A logical conclusion to draw is that a 1,705,608 decrease in industry demand in real world terms is too great to result from a relatively small $2,000 increase in total marketing and R & D. First, a large decrease in demand from a small increase in marketing and R &D would be very rare and secondly, a decrease of more than 50% would be a more than unlikely as a real world event. The authors' example has created the appearance of stability control by reducing the original advertising range of $50,000 - $250,000 used in the 1983 paper to a range of $100 - $1,000. Reducing the scale of advertising has only given the appearance that explosive potential of industry demand has been reduced.

FOURTH PROBLEM: DOES THE MODEL FOR BOTH MARKETING AND ADVERTISING CREATE BELL-SHAPED FUNCTIONS FOR MARKETING AND R & D?

Goosen (2010) states "*The G/P model actually creates a marketing function that is bell-shaped… there is still some doubt that the results are valid.*"

The quote from Goosen is arguing that the marketing literature does not support the possibility that excessive marketing expenditures could decrease a firm's demand. It is true that there is some controversy on whether excess advertising could cause firm demand to decline. As Goosen (2010) states the most accepted relationship is one of diminishing returns but not negative returns. Yet, there is evidence to the contrary. For example, a seminal paper by Dorfman and Steiner (1954) supports the possibility of negative returns to advertising.

While it may be true there exists some research showing that excessive advertising at some point can decrease sales, I would contend that advertising which at some point excessively decreases industry demand is not a feature that should exist in business simulations. The GMS authors then present a new version of the G/P demand equation which they argue does not allow advertising to result in decreasing demand at some level of advertising. This revised model was not presented or discussed in the original G/P 1983 paper. The revised equation is:

$$Q = g1P^{-(.01 + .005)}M^{.5}R^{.5}$$

Concerning this equation, the GMS authors state:

This form of the Gold/Pray function has diminishing returns to marketing and R & D but does not allow for negative returns at any level.

The GMS authors did not present a numerical example of this equation. The claim that this equation has diminishing returns appears to not to be completely true. No data was presented to support this claim; consequently . I set g1 to the value of 10 and then used this equation and the parameters suggested by the GMS authors to generate the following results:

TABLE 4
RESULTS FROM USING REVISED GOLD/PRAY EQUATION

Price	Advertising	R& D	Demand	Increase in Demand	Revenue
20	100	100	719		$14,380
20	200	200	1,439	720	$28,780
20	300	300	2,158	719	$43,160
20	400	400	2,877	719	$57,540
20	500	500	3,596	719	$71,920
20	600	600	4,316	720	$86,320
20	700	700	5,045	719	$100,900
20	800	800	5,754	719	$115,080

Unless I have misunderstood, the GMS authors, it seems that the simplified Gold/Pray industry demand equation based on the parameters they presented creates constant returns rather than diminishing returns. My own calculations using the revised equation as shown in Table 4 indicates that the assignment of .5 as the exponent for M and R create a constant increase in demand. It is apparent that the revised equation prevents negative returns but does not necessarily create diminishing returns as stated by the GMS authors. However, when I assigned exponent values other than .5 such as .4 and .6 to M and R the equation did created diminishing return results.

FIFTH PROBLEM: LACK OF PROOF THAT THE MULTIPLICATIVE MODEL IS DESIRABLE OR SUPERIOR

Goosen (2010) states "The article by Gold and Pray in which their model was presented does not present any rationale as to why the multiplicative model is better. The article simply assumes that it is better."

Choice of words are obviously important and the meaning of a word used by a speaker may not be the same for the hearer. The GMS authors in their reply say:

> The G/P paper began by first outlining the advantages and disadvantages of alternative functional forms for modeling demand including: linear, non-linear, and multiplicative (page 102), and then suggesting a demand system that was argued to possess a number of desirable properties, but was never argued to be superior to all other demand models.

If "superior" is not the correct term then perhaps the term "better" or "less problem oriented" would have been a more suitable word. But the G/P authors in their 1983 paper did say this:

1. *". ... the development of a system of equation which may be employed to model both industry and firm level demand... which will remain stable and does not have the shortcomings found in the other games."*

Clearly this statement implies that the G/P model is better by being stable. Examples of how other models are not stable were not presented.

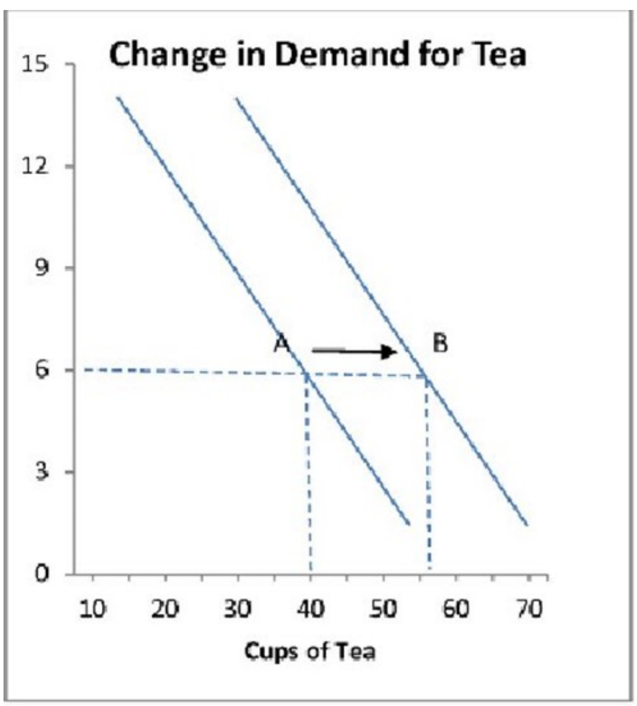
2. *"The paper presents an effective method for modeling and stimulating demand."*

The use of the term "effective" seems to imply that other methods are not always effective. Examples how how other models could be "ineffective" were not presented.

3. *"Eliminating the impact of irrational or faulty decision inputs on total market demand determination."*

Unless I have misread, this statement implies that other types of demand models allow irrational decisions.

FIGURE 2



4. “ The paper presents an approach of modeling an simulating demand which is based on contemporary economic and marketing theory and employs table mathematical functions.”

While this statement does not say that other models are not based on contemporary economic and marketing theory, it seems reasonable that the reader might assume there is a hint being made that other types of models are not necessarily based on sound theory. There is one aspect of the Gold/Pray model that seems to differ from economic theory. In most principles of economic textbooks, the shift resulting an increase in advertising is shown in Figure 2.

In this type of shift regardless of price, the increase in quantity is the same. If this diagram represents current theory, then the Gold/Pray models does not support this type of shift. In terms of a percentages, this shift shown in Figure 2 creates a smaller percentages change in demand at each lower price. As stated earlier, the G/P model creates shifts such that all prices the percentage increase in demand is the same. The illustration above shows that the increase in demand in units is the same at all prices. In other words, if at a price of 100 the increase in demand is 50 units, then at a price of $80 or $60 the increase is also 50 units. While the increase in demand is the same the percentage change at each price is less. Whether a shift in the Gold/Pray demand curve or the one shown in Figure 2 best represents reality or is most desirable in business simulations, as far as I know, has never been discussed or researched in business simulation literature.

SUMMARY AND CONCLUSIONS

Gold and Pray presented their demand algorithm as an improvement over existing demand algorithms commonly found in business simulations. In some respects, this may be true. However, Gold and Pray said:

As noted, certain of the demand functions were somewhat unstable and yielded unrealistic results, if left unconstrained, The designers, in most cases, imposed constraints on the decision variables to prevent discrepancies between theory and simulation play.

If my analysis is correct, then the Gold/Pray demand model can also at times also yield unstable and unrealistic results. If simulation designers find the Gold/Pray useful and want to eliminate negative advertising results for example, then the solution is simply for simulations designers to specify the high and low range for advertising. The same would be true for R & D. A specified range of values for advertising and R & D would also reduce explosive demand results. However, to do this would nullify the GMS author’s statement that the G/P demand functions do not need “imposed restraints.”

The Gold/Pray model as originally presented in 1983 was an important advance in business simulation development. Their paper brought into view weaknesses in several types of demand algorithms. Whether the Gold/Pray model has some weaknesses and problems of its own prior to my own 2010 paper was never seriously addressed. The GMS authors response to my 2010 paper bought some new insights and modification to the Gold/Pray demand system of equations. The authors presented some suggestions on how some of the problems could be eliminated. However, the GMS authors, as I have discussed above, did not resolve the problems I raised. There exists many simulations that one can find on Web sites. Regarding currently existing business simulations, however, there exists no information concerning what type of demand algorithms are being used and also what kind of problems may inherently exists in demand algorithms of these simulations. After thirty five years, the nature of demand equations used in business simulations for the most part are still being kept in a black box. The lids that open this black box still hasn’t been made visible to users and researchers. I agree wholeheartedly with the GMS authors when they encourage “healthy debate on the designing of demand and other functions in computerized business simulations.” Unfortunately, since 1983, little debate and meaningful analysis of strengths and weaknesses of demand algorithms has happened.

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