

HOW TO MIX THE MEMBERSHIP OF GROUPS: A SOLUTION TO FREE RIDING, LEADERSHIP TRAINING, AND EARLY DOMINANCE

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ABSTRACT

The rationale for mixing the membership of groups is presented and the result of mixing is defined for three levels of quality: minimal, complete, and perfect. Two systematic mixing methods, transposition and ladder rotation, are explored. Four findings on the number of complete- and perfect-quality mixes that are possible under specified conditions are explained. Steps for manually implementing transposition and ladder rotation are given, and equations for implementing them computationally are supplied. A table is presented showing the number of complete- and perfect-quality mixes that can be obtained with number of groups ranging from 2 to 7 and group sizes of the same range.

Keywords: complete quality, early dominance, free riding, group membership, ladder rotation, leadership training, minimal quality, mixing, perfect quality, prime number, remixing, transposition

INTRODUCTION

Usually, when elements are mixed to form a compound, the elements cannot be extracted from the compound for a new mix. When students are mixed to form groups, however, the students can be remixed to form new groups. When groups are used repeatedly in conjunction with a single experiential learning exercise, a series of activities, several case studies, or a business simulation game, remixing the membership from round to round hinders free riding (Wolfe & McCoy, 2011), the locus of frequent student complaints about group projects (Glenn, 2009; Lang, 2022); enables more students to practice group leadership skills; and ameliorates the issue of early dominance in team-scored games.

Wang's (2022) role-play game for social sustainability is an exercise where groups are remixed. Participants are grouped by stakeholder roles (i.e., company executive, PR/communication team, company supplier employees, labor activist, investors, and customers) in one round and by company, composed of different stakeholders, in the following round. Even though the exercise takes place over five weeks, the groups are mixed only once. Were it easy to do, mixing the groups additional times would further enrich the game and raise the number of students who would have the experience of leading a group.

More pertinent to leadership training is Hornyak and Page's (2003) leadership training program. The program is limited to 12 participants at a time and composed of four group activities: hula-hoop, spaceship, spider web, and balance board. Although leadership is required to perform each activity successfully, the program apparently does not give every participant the experience of leading a group. If everyone in a program is to have the experience of leading a group in one of the four activities, however, each group must have 4 members, the groups must be mixed three times, and the number of groups should be a prime number greater than three. The prime-number-greater-than-three condition, explained later, means that for groups of 4 members, the number participants should be 20 (5 groups) or 28 (7 groups) or 44 (11 groups), so more than the 12-participant limit of the program.

For case studies and business simulation games, the use of groups fixed in membership for the duration of the course or game is common. In these situations, the instructor desiring to mix the membership of groups from round to round has the added flexibility of varying the number of case studies or decision periods of the game in each round. When learning to lead is among the objectives, the instructor may desire that every student lead a new group for at least one round, thus requiring the mixing of groups between rounds for at least as many rounds as the size of each group.

Even if the instructor opts to allow team members of a business simulation game to select their own leaders, however, mixing the membership of teams from round to round ameliorates the contested issue of early dominance (Bernard & de Souza, 2009; Goosen, 2018; Patz, 1992, 1999, 2000; Peach and Platt, 2000; Rollier, 1992; Teach & Patel, 2007; Wolfe, Biggs, & Gold, 2013), whereby the relative standing of simulated firms in early decision periods strongly affects their relative standing afterwards. For example, the instructor could assign the 20 participants of a classical team-scored business game to five teams of four, run the game for one to three decision periods for the first round, mix the participants, continue the game for another

one to three decision periods for the second round, and so forth through five rounds. Properly mixed, in each round each team will comprise members new to each other and to the team’s assigned firm. The good, or bad, fortune of team composition and assignment will therefore be dispersed to all participants.

Mixing is difficult, however, if the result must be of high quality. Random mixing does not assure results, so systematic methods are necessary. The discussion that follows addresses the quality of the outcome of applying two systematic mixing methods: transposition and laddered rotation.

SYSTEMATIC MIXING METHODS

We begin by defining terms. A mix is a collection of groups for one round of an activity. We assess the quality of a mix relative to other mixes of the set at three levels: minimal, complete, and perfect. We say a mix is of minimal quality if all the members of every group in the mix are not also all the members of a group in another mix. We say a mix is of complete quality if no two members of every group in a minimal mix are not also the same two members of a group in another minimal mix. We say a mix is of perfect quality if no member of every group in a complete mix is also a member of the same group in another complete mix. Thus, a perfect-quality mix is a subset of a complete-quality mix, and a complete-quality mix is a subset of a minimal-quality mix.

A minimal-quality mix is suitable if the instructor’s objective is to highlight the consequences of membership turnover when one member leaves a group and is replaced by an outsider. A complete-quality mix is necessary if the instructor’s objective is to construct new groups for a new mix whose membership is maximally diversified relative to the membership of previous mixes. A perfect-quality mix is required if each group constitutes a team for a business simulation game and the instructor desires to assure that no member of a team has more experience with the team’s simulated firm than any other member.

We proceed to give examples of mixes of different qualities and explain the systematic method that may be applied to achieve each level of quality. We state our findings and show why they are correct. We then present steps for manual implementation, equations for computerized implementation, and a table for selecting group sizes and number of groups that will result in the greatest number of complete-quality and perfect-quality mixes using laddered rotation, a generally applicable systematic mixing method well suited to computerization. We conclude with a summary and suggestions for further research and business-game development.

MIXES OF MINIMAL AND COMPLETE QUALITY

Figure 1 is an example showing 5 mixes of 4 groups, each of which has four members assigned across 4 positions. The members are labeled with alphabetical letters, from A to P. Mixes, groups, and positions are numbered from base 0. Mix 0 is the initial mix. Position numbers are reference points without significance on the status of members.

Mixes 0 through 3 differ only in the assignment of the members in Position 0. These members are rotated one place to the right across groups, so Member A moves from Group 0 in Mix 0 to Group 1 in Mix 1, from there to Group 2 in Mix 2, and so forth, with the other members in Position 0 following along and looping back past Group 3 to Group 0. Mix 0 through Mix 3 are of minimal quality because one member of each group is new in every mix. None of the four mixes are of complete quality, because three members of each group of every mix, occupying Positions 1 through 3, are also the same three members of the same group in another mix.

Mix 4 is created by transposing the groups and positions of Mix 0. It is of complete quality relative to the other four mixes because no two members of any Mix-4 group are also the same two members of a group in another mix. Thus, for example, members A, B, C, and D are members of the same group in Mix 4, but members of different groups in every one of the other

Figure 1

Mixes 0 Through 3 of Minimal Quality and Mix 4 of Complete Quality Relative to Each of the Other Mixes

Position	Mix 0				Mix 1				Mix 2				Mix 3				Mix 4			
	Group				Group				Group				Group				Group			
	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
0	A	B	C	D	D	A	B	C	C	D	A	B	B	C	D	A	A	E	I	M
1	E	F	G	H	E	F	G	H	E	F	G	H	E	F	G	H	B	F	J	N
2	I	J	K	L	I	J	K	L	I	J	K	L	I	J	K	L	C	G	K	O
3	M	N	O	P	M	N	O	P	M	N	O	P	M	N	O	P	D	H	L	P

mixes. Still, Mix 4 is not a perfect mix, because three members of the diagonal (F, K, and P, shaded) are also members of their same respective groups in every one of the other mixes.

LADDERED ROTATION

A solution to the limitation of rotating only the members of Position 0 is to rotate the members of all positions in ladder fashion, such that members of Position 1 are rotated two places, members of Position 2 are rotated 3 places, and so forth. The result on Mix 0 of Figure 1 is shown in Figure 2. As shown, ladder rotation upgrades the quality of Mix 1 from minimal to complete, because no two members of a group in Mix 1 are also the same two members of a group in Mix 0. Mix 2 and Mix 3, however, remain of minimal quality relative to Mix 0 and Mix 1, respectively, because the member assignments of Position 3 are unchanged throughout and member assignments of Position 1 for Mix 2 and Mix 3 revert to their assignments of Mix 0 and Mix 1, respectively. Thus, four member-pairs of Mix 2 (E-M, F-N, G-O, and H-P, shaded) are likewise paired in Mix 0 and four member-pairs of Mix 3 (G-M, H-N, E-O, and F-P, shaded) are likewise paired in Mix 1. In this instance, ladder rotation does not give rise to complete quality beyond Mix 1.

Figure 2
Mixing Group Membership by Laddered Rotation

Position	Mix 0				Mix 1				Mix 2				Mix 3			
	Group				Group				Group				Group			
	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
0	A	B	C	D	D	A	B	C	C	D	A	B	B	C	D	A
1	E	F	G	H	G	H	E	F	E	F	G	H	G	H	E	F
2	I	J	K	L	J	K	L	I	K	L	I	J	L	I	J	K
3	M	N	O	P	M	N	O	P	M	N	O	P	M	N	O	P

FINDINGS

The examples of Figure 1 and Figure 2 are instances where the size of every group is the same. The equal-size constraint simplifies argument without undermining generality, for dummy members can always be added to equalize sizes without changing conclusions. Our findings are as follows:

1. For any number of groups, n , the number of complete-quality mixes that are possible of groups sized greater than n is 1 and the number of complete-quality mixes that are possible of n -sized groups is $n + 1$.
2. When n is a prime number, the number of complete-quality mixes that are possible with ladder rotation of groups sized n or fewer is n .
3. When n is a prime number, the number of perfect-quality mixes that are possible with ladder rotation of groups sized fewer than n is n .
4. When n is not a prime number, n can be divided into two factors, a and b , such that a is the smallest prime factor of n . Then the number of perfect-quality mixes that are possible with ladder rotation of groups sized b or more but fewer than n is a , and the number of complete mixes that are possible with ladder rotation of groups sized $2a$ or more but no more than n is b .

FINDING 1

For a mix to be of complete quality, no more than a single member of any group can remain in that group after the members are mixed. But if the size of a group exceeds the number of groups, then the members of the group numbering in excess of group size over group number cannot be placed in a group of a new mix in which no other member of the initial mix has already been placed. Since a new mix of complete quality cannot be constructed, the number of complete-quality mixes reduces to that of the initial mix, 1.

For a proof that the number of complete-quality mixes that are possible of n -sized groups is $n + 1$, consider the equality of Equation 1. Thus, n^2 members distributed evenly across n groups each of n size sum to the same number as $n + 1$ groups of one fewer member in each group, plus one holdout member. Constructing a mix of complete quality requires that the holdout member be added to one of the reduced-sized ($n - 1$) groups to form the desired n -sized group and that the reduced-sized group chosen must differ from one mix to another. There being $n + 1$ unique reduced sized groups in a population of n^2 members, the finding is proved.

Equation 1

Equality

$$n^2 = (n + 1)(n - 1) + 1 \tag{1}$$

FINDING 2

Figure 2 shows why n must be a prime number for the number of complete-quality mixes that are possible with laddered rotation of n -sized groups to be n . As previously noted, all members of Position 1 are in the same group as all members of Position 3 in both Mix 0 and Mix 2, so the number of complete-quality mixes of the 4-mix set is 2, not 4.

The members of Position 1 loop back in Mix 2 to their initial groups in Mix 0 because they are rotated 2 places. Inasmuch as 2 is a factor of 4, the shift returns the members to their initial groups in fewer than $n = 4$ mixes. When n is a prime number, however, only the members of the last position loops back to their initial positions in fewer than n mixes, which they do in every mix. The finding is proved.

FINDING 3

Reiteration the last point, laddered rotation causes the members of the last position, Position $n - 1$, to be fixed to their groups because they loop back to their initial groups with every mix. Reducing the group size by one removes those members, thus upgrading the quality of mixes from complete to perfect. The finding is proved.

FINDING 4

Figure 3 shows laddered rotation applied to the first four mixes of $n = 6$ groups, Greek letters representing members following the last of the Roman alphabets. The factors of 6 are $a = 2$ and $b = 3$. The members of Position 2 are rotated right 3 places at a time, which causes them to loop back to their initial groups by Mix 2, after having been shifted 2 times ($3 \times 2 = 6$). Thus, for groups sized 3 through $n - 1 = 5$, corresponding to Position 2 through Position 4, the factors of 6 reduce the number of possible perfect mixes from $n = 6$ to $a = 2$, as the first part of Finding 4 asserts.

For the second part of Finding 4, The members of Position 1 are rotated right $a = 2$ places at a time, which causes them to loop back to their initial groups by Mix 3, after having been shifted 3 times ($2 \times 3 = 6$). The members of Position 3 are shifted right $2a = 4$ places at a time, which causes them to loop back to their initial groups by Mix 3 also ($4 \times 3 = 12 = 6 \times 2$). Consequently, members of Position 1 are paired with members of Position 3 in Mix 3 as they were in Mix 0. Thus, Mix 3 is not a complete mix relative to Mix 0. Only the $b = 3$ mixes preceding Mix 3 are complete mixes. The result applies to groups sized $2a = 4$ through $n = 6$, corresponding to Position 3 through Position 5, as the second part of Finding 4 asserts.

Figure 3
Laddered Rotation of Six Groups

	Mix 0	Mix 1	Mix 2	Mix 3
	Group	Group	Group	Group
	0 1 2 3 4 5	0 1 2 3 4 5	0 1 2 3 4 5	0 1 2 3 4 5
0	A B C D E F	F A B C D E	E F A B C D	D E F A B C
1	G H I J K L	K L G H I J	I J K L G H	G H I J K L
2	M N O P Q R	P Q R M N O	M N O P Q R	P Q R M N O
3	S T U V W X	U V W X S T	W X S T U V	S T U V W X
4	Z α β γ δ ε	α β γ δ ε Z	β γ δ ε Z α	γ δ ε Z α β
5	ζ η θ ι κ λ	ζ η θ ι κ λ	ζ η θ ι κ λ	ζ η θ ι κ λ

IMPLEMENTATION

Table 1 shows possible number of complete- and perfect-quality mixes by number of groups and group sizes with laddered rotation. The shaded cells are those where group numbers are prime numbers. The instructor can use the table to select the best combination of group number and group size for class sizes from 4 (2×2) to 49 (7×7), compromising as necessary between mix quality and same-size groups. For example, for a class of 26 members, the instructor would choose between 6

groups of 4 to 5 members and accept that only 3 mixes can be of complete quality and only 2 of those can be of perfect quality, or 7 groups of 3 to 4 members for 7 possible perfect-quality mixes. For class sizes exceeding 49, the instructor could divide the class into two or more sections before applying the ladder rotation method to each section independently.

Table 1
Possible Number of Complete- and Perfect-Quality Mixes
by Number of Groups and Group Size With Laddered Rotation

Highest Group Size	Number of Groups					
	2	3	4	5	6	7
2	$\frac{2}{1}$	$\frac{3}{3}$	$\frac{4}{2}$	$\frac{5}{5}$	$\frac{6}{3}$	$\frac{7}{7}$
3		$\frac{3}{1}$	$\frac{2}{2}$	$\frac{5}{5}$	$\frac{6}{2}$	$\frac{7}{7}$
4			$\frac{2}{1}$	$\frac{5}{5}$	$\frac{3}{2}$	$\frac{7}{7}$
5				$\frac{5}{1}$	$\frac{3}{2}$	$\frac{7}{7}$
6					$\frac{3}{1}$	$\frac{7}{7}$
7						$\frac{7}{1}$

Note: The number of complete-quality mixes appears above the bar; the number of perfect-quality mixes appears below the bar. The number count includes the initial mix, so the count of 1 means that mixing the membership to achieve the desired quality is impossible. Empty cells are where a complete-quality mix is impossible, so a perfect-quality mix also is impossible.

As for transposition, the method is easy to execute manually. Thus, the students of each group would be told to self-assign themselves to a position, after which those in each position would meet to form their new groups.

For computer assignment of members to groups by transposition, let $x_{i,j,k}$ refer to a member of mix i who is assigned to position j of group k , one of n groups. Then for any initial assignment of members to groups, $i = 0$, proceed according to Equation 2, where $i + 1$ refers to the transposed mix.

Equation 2
Transposition

$$x_{i,j,k} \Rightarrow x_{i+1,k,j} \tag{2}$$

For ladder rotation, the manual steps are as follows:

1. Construct a table whereby the columns refer to groups and the rows refer to each member's position within each group.
2. For the initial mix, distribute members evenly across the chosen number of groups, added dummy members as necessary to assure that all groups are of the same size.
3. For the next mix, displace every member in the first position to the next column on the member's right, looping the last member back to the first row. Thus, if the five members in the first position are arranged in A-B-C-D-E order, as shown in Figure 4, the displacement will result in the members ordered E-A-B-C-D.
4. Displace every member in the second position to the second column on the member's right, looping members displaced past the last column back through the first column. Thus, if the five members in the second position are ordered F-G-H-I-J, the displacement will result in the members ordered I-J-F-G-H.
5. Proceed likewise to displace every member in the third position to the third column on the member's right, fourth position to fourth column, and so forth until and including the members of the last position.
6. Repeat steps 3 through 5 for the third and subsequent mixes for as many mixes as desired.

FIGURE 4
Mixing the Membership of Five Groups by Laddered Rotation

Position	Mix 0					Mix 1					Mix 2					Mix 3					Mix 4				
	Group					Group					Group					Group					Group				
	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
0	A	B	C	D	E	E	A	B	C	D	D	E	A	B	C	C	D	E	A	B	B	C	D	E	A
1	F	G	H	I	J	I	J	F	G	H	G	H	I	J	F	J	F	G	H	I	H	I	J	F	G
2	K	L	M	N	O	M	N	O	K	L	O	K	L	M	N	L	M	N	O	K	N	O	K	L	M
3	P	Q	R	S	T	Q	R	S	T	P	R	S	T	P	Q	S	T	P	Q	R	T	P	Q	R	S
4	U	V	W	X	Y	U	V	W	X	Y	U	V	W	X	Y	U	V	W	X	Y	U	V	W	X	Y

As to laddered rotation in equation form, for any initial assignment of members to groups, $i = 0$, proceed according to Equation 3, where $i + 1$ refers to the next mix.

Equation 3
Laddered Rotation

$$x_{i,j,k} \Rightarrow x_{i+1,j,(k+j+1) \bmod n} \tag{3}$$

Indexing from base 0 for the first mix ($i = 0$) of five groups ($n = 5$), the member in the third position ($j = 2$) of the first group ($k = 0$) will be shifted to the same position of the fourth group, that is, Group $(k + j + 1) \bmod 5 = (0 + 2 + 1) \bmod 5 = 3$ (Equation 4). Member K is the Position 2 member of Figure 4 who is assigned from Mix 0, Group 0 ($x_{0,2,0}$), to Mix 1, Group 3 ($x_{1,2,3}$).

Equation 4
Laddered Rotation Example

$$x_{0,2,0} \Rightarrow x_{1,2,(0+2+1) \bmod 5} = x_{1,2,3} \tag{4}$$

CONCLUSION

Depending on group size, the number of groups, and the mixing method that is applied, mixes may be of minimal, complete, or perfect quality. As random mixing cannot assure results, systematic mixing is necessary. Laddered rotation is a systematic mixing method more generally applicable than transposition, which although easier to execute manually is limited to two mixes. Transposition loses its ease-of-execution advantage when the mixing process is computerized. Yet, transposition, but not laddered rotation, can be implemented for a mix of complete quality when the number of groups differs between two mixes, as in Wang’s (2022) two-round exercise if 18 participants are divided into 6 groups in the first round and remixed into 3 groups in the second round. Even so, transposition restricts the number of groups after mixing to the size of the largest group before mixing.

Further research should clarify how mixing the membership of groups affects free riding and participants’ learning. Free riding on team assignments where summative peer evaluations (Scherpereel, 2010) have substantial weight on credit towards grades may depend on social contracts that mixing disrupts, but additional studies will be necessary to substantiate the conjectured dependence. Mixing gives more participants more experiences with different roles, especially the group-leadership role, but each participant will have less time in each role. Experimentation with different lengths of rounds, in number of cases or decision periods, will be necessary to determine how much time participants should have with each role before they are switched by mixing to another role for the optimum effect on learning.

Mixing the membership of groups disperses the effect of early dominance on the scores of participants, at some increase in the difficulty of administering a business simulation game. The increased difficulty, however, should be small when the mixing process is computerized, and likely negligible when the mixing is part of a computerized business game.

That mixing the membership of groups addresses three extensively discussed issues of business simulations and experiential learning simultaneously points to its potential to be a valuable tool for active learning and a suitable target for research. We should be at the point where researchers of business simulations and experiential learning cease to be defensive about the virtues of active-learning pedagogies and begin to explore how active-learning pedagogies might be improved.

REFERENCES

- Bernard, R. R. S., & de Souza, M. P. (2009). Dominance in online business games competitions. *Developments in Business Simulation & Experiential Learning, 36*, 287-294. Available from the *Bernie Keys Library*, <https://absel.org>
- Glenn, D. 2009, June 8. Students give group assignments a failing grade. *The Chronicle of Higher Education*. Available <http://chronicle.com/article/Students-Give-Group/47303/>
- Goosen, K. R. (2018). The effect of simulation starting decisions and optimum decisions on firm profit dominance. *Developments in Business Simulation & Experiential Learning, 45*, 97-104. Available from the *Bernie Keys Library*, <https://absel.org>
- Hornyak, M. J. & Page, D. (2003). Experiential learning: Introducing faculty and staff to a university leadership development program. *Developments in Business Simulation & Experiential Learning, 30*, 83-89. Available from the *Bernie Keys Library*, <https://absel.org>
- Lang, J. M. (2022, June 17). Why students hate group projects (and how to change that). *The Chronicle of Higher Education*, Advice. Available from https://www.chronicle.com/article/why-students-hate-group-projects-and-how-to-change-that?utm_source=Iterable&utm_medium=email&utm_campaign=campaign_4573382_nl_Academe-Today_date_20220630&cid=at&source=ams&sourceid=
- Patz, A. L. (1992). Personality bias in total enterprise simulations. *Simulation & Games, 23*, 45-76.
- Patz, A. L. (1999). Overall dominance in total enterprise simulation performance. *Developments in Business Simulation & Experiential Learning, 26*, 115-116. Available from the *Bernie Keys Library*, <https://absel.org>
- Patz, A. L. (2000). One more time: Overall dominance in total enterprise simulation performance. *Developments in Business Simulation & Experiential Learning, 27*, 254-258. Available from the *Bernie Keys Library*, <https://absel.org>
- Peach, E. B., & Platt, R. G. (2000). Total enterprise simulations and optimizing the decision set: Assessing student learning across decision periods. *Developments in Business Simulation & Experiential Learning, 27*, 242-247. Available from the *Bernie Keys Library*, <https://absel.org>
- Rollier, B. (1992). Observations of a corporate facilitator. *Simulation & Gaming, 23*, 442-456.
- Scherpereel, C. M. (2010). Peer to peer: A new tool for student peer evaluation. *Developments in Business Simulation & Experiential Learning, 37*, 238-250. Available from the *Bernie Keys Library*, <https://absel.org>
- Teach, R., & Patel, V. (2007). Assessing participant learning in a business simulation. *Developments in Business Simulation & Experiential Learning, 34*, 76-84. Available from the *Bernie Keys Library*, <https://absel.org>
- Wang, R. T-C (2022). Investigating the contribution of role-plays for social sustainability: Designing a role-play game for CSR communication. *Developments in Business Simulation & Experiential Learning, 49*, 47-52. Available from the *Bernie Keys Library*, <https://absel.org>
- Wolfe, J., Biggs, W. D., & Gold, S. C. (2013). Early-determined business endgame standing: A replication and expansion of Teach and Patel's findings. *Simulation & Gaming, 44*, 493-513.
- Wolfe, J. & McCoy, R. (2011). Those who do and those that don't: A study of engaged and disengaged business game players. *Developments in Business Simulation & Experiential Learning, 38*, 383-389. Available from the *Bernie Keys Library*, <https://absel.org>