EVALUATION OF COLLABORATIVE FILTERING BY AGENT-BASED SIMULATION CONSIDERING MARKET ENVIRONMENT

Takashi Umeda
Tokyo Institute of Technology
umeda,ichikawa@cs.dis.titech.ac.jp

Manabu Ichikawa
Tokyo Institute of Technology

Yuhsuke Koyama
Tokyo Institute of Technology

Hiroshi Deguchi
Tokyo Institute of Technology
koyama,deguchi@dis.titech.ac.jp

ABSTRACT

We propose a new evaluation approach for collaborative filtering, a kind of recommendation algorithm through agent-based simulation. We modeled a virtual E-commerce market where we evaluated the collaborative filtering algorithm. Our findings were as follows: 1) the number of neighbors is a key parameter and there is a trade-off due to market circumstances, 2) a bigger number of neighbors performed better, with a tendency that was independent of the degree of clustering of consumer preferences, 3) if there were any high-frequency purchasers, a smaller number of neighbors performed better.

INTRODUCTION

Electronic-commerce (EC) websites have been increasing in number and market size around the world. In 2007, Japan's business to consumer EC market size amounted to 5.3 trillion yen (Ministry of Economy, Trade and Industry 2008). Some EC websites employ a system for recommending items to consumers, but these systems might not be working effectively. To optimize the system parameters, we need some virtual evaluation environments.

In this paper, we propose a simulation model for evaluating and designing recommendation algorithms considering the market environment. This model is a business simulation able to provide useful knowledge for people engaged in web marketing, web systems or EC business. As an application of the model, evaluation and parameter optimization is performed for a user-based approach that is a kind of collaborative filtering algorithm, the popular recommendation algorithm used on many websites.

METHODOLOGY OF THIS RESEARCH

Recommendation algorithms are usually evaluated by cross-validation or human-subject experiments.

In cross-validation (Sarwar 2001), data is divided into a test portion and a training portion and researchers measure the error between the test data and the estimated values computed with the training portion. This method is convenient to use when there is sufficient available data. However, it is difficult to evaluate the recommendation algorithms considering the market environment in cases where consumer behavior is very complex in accordance with their attributes, characteristics, and taste in items. It is even impossible to use this method when the market environment changes frequently, for example, new items are dynamically and constantly added and consumer preferences dynamically change according to the website recommendations.

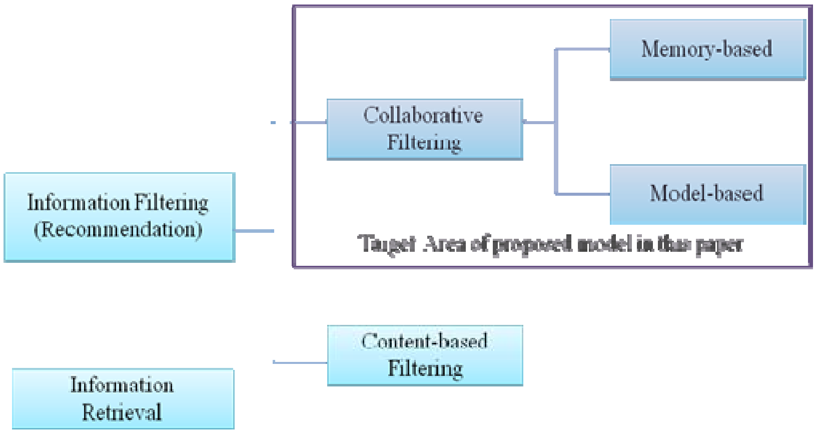
A human-subject experiment (Zhang 2002) is a good approach for evaluating algorithms considering the market environment. However, an experimental trial takes a long time and it is not easy to attract many subjects.

Consequently, an appropriate evaluation method is needed. We propose a new evaluation method, an agent-based simulation (ABS). The ABS method has the advantage of modeling agent heterogeneity and the interaction among agents. We created a model containing recommendation algorithms, consumer decision-making algorithms, item characteristics, and their relation to consumer preferences. This model is able to estimate the optimal parameter settings in the algorithms and the accuracy of recommendations considering the market environment and changes in the environment. Furthermore, in cases where it is difficult to obtain past purchase data, researchers can easily evaluate the recommendation algorithms under various assumptions. Table 1 summarizes the characteristics of the evaluation methodologies.

Table 1 Comparison of methodologies

Methodology	Ease of use	Market environment	Changes in environment
Cross-validation	OK	NG	NG
Human-subject experiments	NG	OK	Depends
ABS	OK	OK	OK

Figure 1 Category of recommendation



RELATED WORK

RECOMMENDATION ALGORITHMS

In this section, we briefly present some of the research related to information filtering and recommender systems. Recommender systems can be broadly categorized into two types: content-based filtering and collaborative filtering. In content-based filtering, recommendations are made by using information about the item, such as the category, tag, and description. When using this method, additional information on the item must be obtained manually. So, it would be difficult to apply content-based filtering when there are items for which additional information is not supplied. In collaborative filtering, the recommender system uses the item ratings by users to generate recommendations, and it is typically content agnostic. So, it is advantageous to recommend a set of items from a large number of items regardless of whether or not additional information is supplied. The model that we propose is useful for evaluation and parameter optimization for collaborative filtering.

Collaborative filtering systems can be further categorized into memory-based and model-based algorithms. memory-based algorithms generate ratings predictions for users based on their past ratings. In addition, the memory-based method is categorized into user-based (Resenick 1994; Herlocker 1999) and item-based (Sarwar 2001) methods. The former is calculated as the weighted average of the ratings given by other users where the weight is proportional to the similarity between users. The latter method is calculated as the weighted average of the ratings given by the subject user where the weight is proportional to the similarity between items. In contrast to memory-based algorithms, model-based algorithms try to model the users based on their past ratings and then use these models to predict the ratings on unseen items. Examples of this approach are methods using the Bayesian network (Breese 1998), LSH (Abhinandan 2007), probabilistic latent semantic indexing (pLSI) (Hofman 1999; Hofman 2004), the Markov decision process (Shani 2002; Shani 2005) and clustering (Breese 1998). Both the memory-based and model-based approach are used for business applications, for example, on the online retail book store, Amazon.com (Linden 2003) and the online news website, Google News (Abhinandan 2007).

Our model can be applied to various approaches for the collaborative filtering mentioned above. In this paper, we focus on the “user-based approach”, which is the basic one in collaborative filtering algorithms. Evaluation and parameter optimization is performed.

In introducing and deploying a recommender system using collaborative filtering, some problems have been pointed out, namely in terms of cold start and gray sheep. The cold-start problem is a phenomenon whereby the quality of recommendations deteriorates soon after introducing the recommender system due to insufficient rating data (Balabanovic 1997). In the gray sheep problem, the recommender system is unable to classify some users due to their unusual preferences (Burke 2002). Later, we duplicate the above market environment, and analyze these two problems in the user-based approach.

CONSUMER BEHAVIOR ON THE E-COMMERCE WEBSITE

In economics and marketing science, research is conducted on consumer behavior on E-commerce websites.

Figure 3 Overview of model

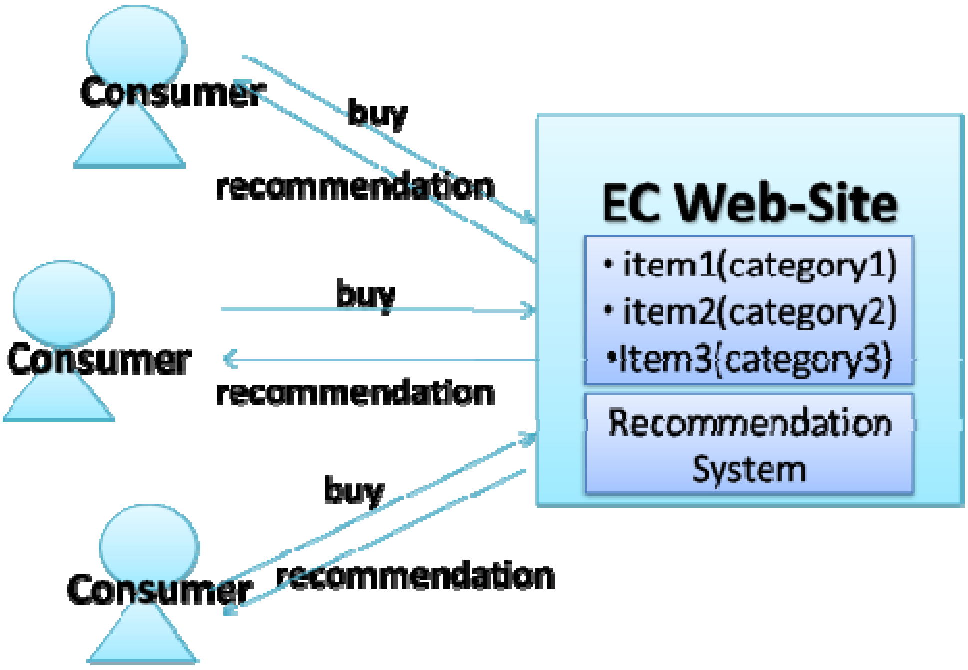
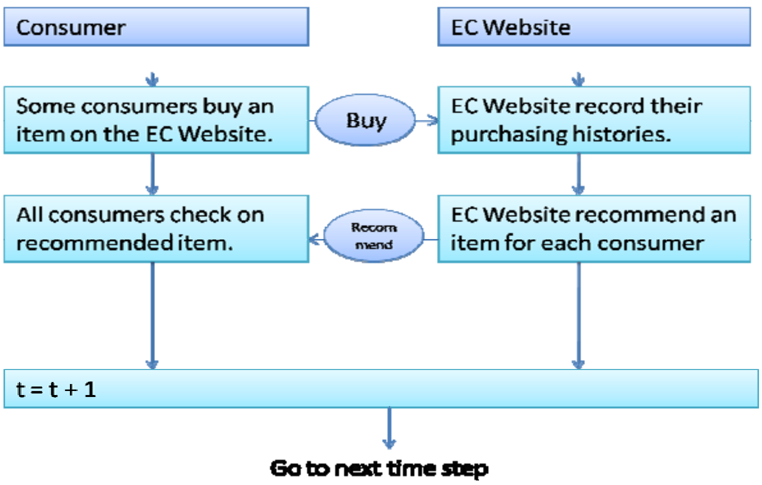


Figure 3 Simulation process



Nelson (1970) suggests that goods can be classified as either search products or experience products. Search products are those that "the consumer can evaluate by inspection prior to purchase", for example, PCs and cameras. Experience products are those that "are not evaluated prior to purchase", for example, books, CD/DVDs and movies. King (1994) found that consumers assessing a search product are more likely to use own-based decision-making process compared to consumers assessing an experience product, and that consumers evaluating an experience product rely more on other-based and hybrid decision-making processes compared to consumers assessing a search product. Furthermore, it was verified that recommendations for experience products were significantly more influential than recommendations for search products (Senecal 2004).

According to the findings of one survey, it is effective to make recommendations for books, software, and CD/DVDs, but not effective for home electronics (Hotlink 2008).

In this paper, we focus on recommendations using collaborative filtering for experience products such as books, CD/DVDs, movies, etc.

MODEL

OUTLINE OF THE MODEL

Our model contains a virtual EC website and several consumers (Figure 2). The EC website uses the recommendation system. This system records the entire purchasing history for every consumer and makes recommendations on items for each consumer. Each item sold on the EC website belongs to one category. Each consumer has a preference for each category as well as several parameters that are used to decide what he buys.

In each time step, consumers are given recommendations on items from the EC website in accordance with their preferences or past recommended items and some of the consumers buy an item (Figure 3).

ASSUMPTIONS OF THE MODEL

This model makes the following assumptions:

- An item belongs to one category. An item cannot belong to more than two categories.
- Items in this model are experience products, for example, books, software and movies. This model cannot be applied for search products because consumer preferences and item attributes in the search product market must be modeled differently from the case of experience products.
- Recommendations are generated by collaborative filtering. Content-based filtering is not intended.
- The category preference of each consumer is initially given and is not dynamically changed. However, customer evaluation of each item can be dynamically changed.

ABSTRACT DEFINITION OF THE MODEL

- **Item**

A set of times is defined as follows:

$$TS = \{1,2,\ldots,q\}$$

The EC website has a set of items defined as follows:

$$IS = \{1,2,\ldots,l\}$$

A set of categories is defined as follows:

$$CS = \{1,2,\ldots,m\}$$

Each item $i \in IS$ belongs to a category and the category of an item $i \in IS$ is defined as IC_i. The number of items $i \in IS$ sold until time t is defined as $sold_{i,t}$.

- **Consumer**

A set of consumers is defined as follows:

$$US = \{1,2,\ldots,n\}$$

Consumer has a preference for each category and the preference of consumer $u \in US$ for category $c \in CS$ is defined as follows:

$$pf_{u,c} = \begin{cases} 1, \text{if consumer } u \in US \text{ is interested in category } c \in CS \\ 0, \text{if consumer } u \in US \text{ is not interested in category } c \in CS \end{cases} \quad (1)$$

This value is given initially. The preference list of consumer $u \in US$ is defined as follows:

$$PF_u = (pf_{u,1}\ pf_{u,2}\ pf_{u,3} \ldots\ldots pf_{u,m})$$

The decision by consumer $u \in US$ to buy an item at time t is defined as follows. “null” represents not buying an item.

$$\mathrm{Decf}: US \times TS \rightarrow IS \cup \{\mathrm{null}\} \quad (2)$$

The probability that Decf (u, t) is an element of IS is represented by $prob_u$ and is called “purchasing probability of consumer u”. In this model, the size of the purchase volume is represented not by the size of purchase volume per term but by the purchasing probability.

A set of items bought by consumer $u \in US$ until time t is defined as follows:

$$BI_{u,t} = \{\mathrm{Decf}(u,j) \mid j = 1,2,3,\ldots,t\}$$

A set of items that is not bought by consumer $u \in US$ at time $t \in TS$ is defined as follows:

$$NI_{u,t} = IS \setminus BI_{u,t} \quad (3)$$

- **Recommendation system**

The rating value for item i by consumer u at time t is defined as follows:

$$v^t_{u,i} = \begin{cases} 1 \ , \text{if} \ \ i \in BI_{u,t} \\ 0 \ , \text{if} \ \ i \notin BI_{u,t} \end{cases} \quad (4)$$

The system recommends the set of items for which the estimated rating is high at every term. We defined collaborative filtering as the recommendation method using not the metadata, for example, the item category, but rather the rating data by the consumer. Also, we distinguish collaborative filtering from content-based filtering. Here, the recommendation system performing collaborative filtering is defined as follows:

$$\mathrm{Recf}: US \times TS \rightarrow \wp(NI_{u,t}) \quad (5)$$

CONCRETE DEFINITION OF THE MODEL

- **Recf – Recommendation model**

In *Recf*, the collaborative filtering algorithm, especially for user-based collaborative filtering (Herlocker 1999), is modeled for the recommendation model. We define $w^t_{u,j}$ as the similarity between consumer u and consumer j. $w^t_{u,j}$ is computed as follows. This is the cosine of two vectors that represent the rating of items by consumer u and j.

$$w^t_{u,j} = \sum_{i \in IS} \frac{v^t_{u,i}}{\sqrt{\sum_{s \in IS}(v^t_{u,s})^2}} \frac{v^t_{j,i}}{\sqrt{\sum_{k \in IS}(v^t_{j,k})^2}} \quad (6)$$

We define $NE_{u,ne}$ as the set of top-*ne* consumers who have preferences similar to consumer u. The user contained in $NE_{u,ne}$ is called the *neighbor of u* and *ne* is the *number of neighbors*. The estimated rating of consumer u for the non-purchased item $i \in NI_{u,t}$,$ev^t_{u,i}$, is computed as follows:

$$ev^t_{u,i} = \sum_{j \in NE_{u,ne}} w^t_{u,j} v^t_{j,i} \quad (7)$$

The recommended items for consumer u at time t are the set of top-r items that have a high estimated rating value, $ev^t_{u,i}$. r and *ne* are operational parameters.

- **Decf – Decision making on what to purchase by consumer**

Each consumer purchases an item with probability $prob_u$ and does not purchase an item with probability 1-$prob_u$. In the case of purchasing an item, each consumer selects the item from the non-purchased items, $NI_{u,t}$. We defined $eval_{u,t}(i)$ as the evaluation value of consumer *u* for the item, computed as follows. This value is computed for all items not purchased by consumer u,$NI_{u,t}$.

$$\mathrm{eval}_{u,t}(i) = w1\frac{\mathrm{pf}_{u,IC_i}}{\sum_{j\in NI_{u,t}} \mathrm{pf}_{u,IC_j}} + w2\frac{\beta_{i,t}}{\sum_{j\in NI_{u,t}} \beta_{j,t}} + w3\frac{\mathrm{sold}_{i,t}}{\sum_{j\in NI_{u,t}} \mathrm{sold}_{j,t}} \tag{8}$$

Where

$$w1, w2, w3 \in [0,1], w1 + w2 + w3 = 1$$

$$\beta_{i,t} = \begin{cases} 1, \text{if } i \in \mathrm{Recf}(u, t-1) \\ 0, \text{if } i \notin \mathrm{Recf}(u, t-1) \end{cases}$$

w1, *w2* and *w3* are elements of the criteria for decision making on which item to buy. *w1* is the degree of emphasis on the category of an item, *w2* is the degree of emphasis on a recommendation from the EC website, and *w3* is the degree of emphasis on the popularity of an item. Based on these evaluation values, the probability that consumer *u* will buy item $i \in NI_{u,t}$ at time t, $P_{u,t}(i)$, is defined as follows:

$$P_{u,t}(i) = \frac{\mathrm{eval}_{u,t}(i)}{\sum_{j\in NI_{u,t}} \mathrm{eval}_{u,t}(j)} \tag{9}$$

EVALUATION METRICS FOR THE RECOMMENDATION SYSTEM

In evaluating the recommendation algorithms, "precision" and "recall" are usually used as evaluation metrics (Herlocker 1999). In other cases, novelty, serendipity and diversity are proposed as evaluation metrics (Herlocker 2004). These metrics are mainly used in human-subject experiments, but they cannot be applied for simulation-based methodology. So, in this research, we use "precision" for the evaluation metrics.

Here, a set is defined in order to define the evaluation metrics. FI_u is a set of items favored by consumer *u*.

$$FI_u = \{i \mid i \in IS \wedge pf_{u,IC_i} = 1\} \tag{10}$$

Precision is defined as the number of times a consumer expresses interest in the recommended items. Precision for consumer *u* at time *t* is defined as follows:

$$\mathrm{precision}_{u,t} = \frac{|\mathrm{Recf}(u,t) \cap \mathrm{FI}_u|}{|\mathrm{Recf}(u,t)|} \tag{11}$$

SETTING OF PARAMETERS

The common parameters for each market environment are shown in Table 2.

FREQUENCY OF PURCHASING: 2 CASES

- *Homogeneous*: The parameter *prob* called '*probability of purchasing*' for each consumer agent is 0.5.
- *Heterogeneous*: There are 20% "high-frequency purchasers" and 80% "low-frequency purchasers". The parameter *prob* for high-frequency purchasers is 0.8 and that for low-frequency purchasers is 0.2.

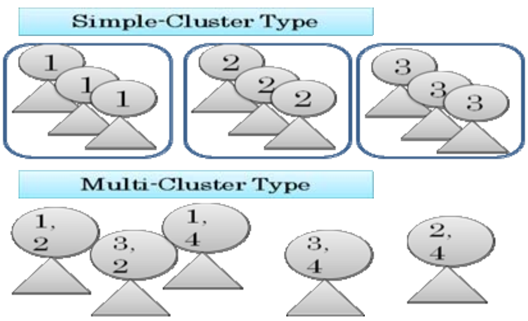
PURCHASING CRITERIA: 2 CASES

- *Preference-conscious type*: The parameter (*w1*,*w2*,*w3*) for each consumer agent is (0.9,0.1,0.0)
- *Trend-chasing type*: The model contains some trends and trend chasers. The parameter (*w1*, *w2*, *w3*) for a trend-chaser is (0.5,0.1,0.4) and that for the rest is (0.9,0.1,0.0). We assume that the share of trend-chasers is 30%. Such agents are likely common in the retail

Table 2 Setting of parameters

Parameter		Value
q	*Time period*	50
m	*Number of categories*	10
$prob_u$	*Frequency of purchasing*	{*homogeneous*, *heterogeneous*}
(*w1*,*w2*,*w3*)	*Purchasing criteria*	{ *preference-conscious type*, *trend-chasing type*}
	Distribution of preferences	{*Simple-cluster type*, *Multi-cluster type*}
ne	*Number of neighbors*	{5,15,25,35,45}
l	*Number of items*	2000
n	*Number of consumers*	1000
r	*Number of recommended items*	10

Figure 4 'Simple-cluster' and 'Multi-cluster' types (numbers indicate the categories that each agent likes)



music market.

DISTRIBUTION OF PREFERENCES: 2 CASES

- *Simple-cluster type*: In this case, consumer preferences are highly clustered. For example, there is a group consisting of consumer agents that like Category 1, a group consisting of consumer agents that like Category 2,….as shown in Figure 4
- *Multi-cluster type*: Consumer agents like two categories at random as shown in Figure 4. In this situation, we can't cluster consumers and find similar consumers. This is called "*Gray sheep problem*" (Burke 2002).

We selected four important market environments (Table 3). In each market environment, we show the results from changing the value of *ne* (number of neighbors). Note that this parameter has a significant effect on the quality of recommendation (Balabanovic 1997).

EXPERIMENTAL RESULTS

EFFECT OF DISTRIBUTION OF PREFERENCES

Figures 5 and 6 show the time evolution for the average precision in market environment 1 and 2 where we changed the number of neighbors, *ne*, from 5 to 45 in increments of 10. From these figures, we can see that precision is independent of *ne* when *distribution of preferences* is the simple-cluster type. Whereas precision is high at *ne* = 25, 35, and 45 when *distribution of preferences* is the multi-cluster type. It can also be seen from the results that when the *distribution of preferences* is the multi-cluster type, the precision value is lower than that when the *distribution of preferences* is the simple-cluster type.

Table 3 Setting of scenarios

Market environment	Distribution of preferences	Frequency of purchasing	Purchasing criteria
1	*Simple-cluster type*	*homogeneous*	*preference-conscious*
2	*Multi-cluster type*	*homogeneous*	*preference-conscious*
3	*Simple-cluster type*	*heterogeneous*	*preference-conscious*
4	*Multi-cluster type*	*heterogeneous*	*preference-conscious*

Figure 6 Market environment 1 (average precision calculated for all consumers)

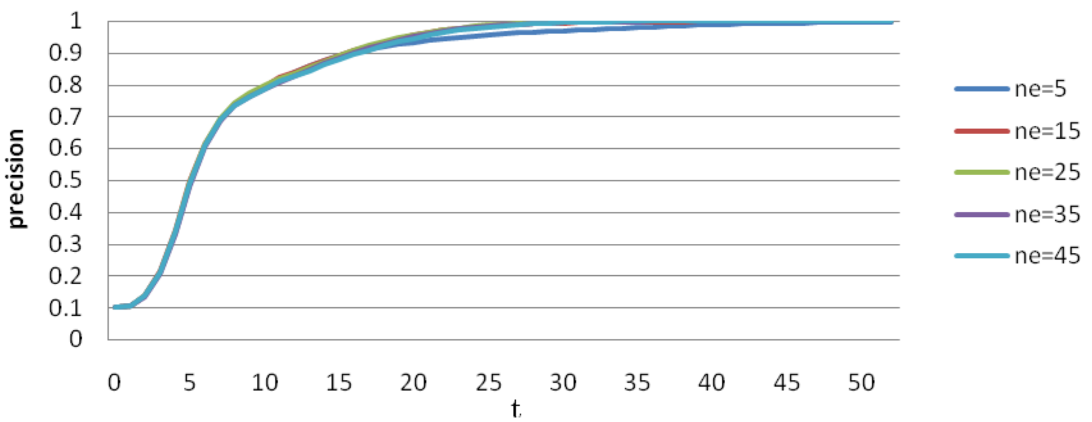
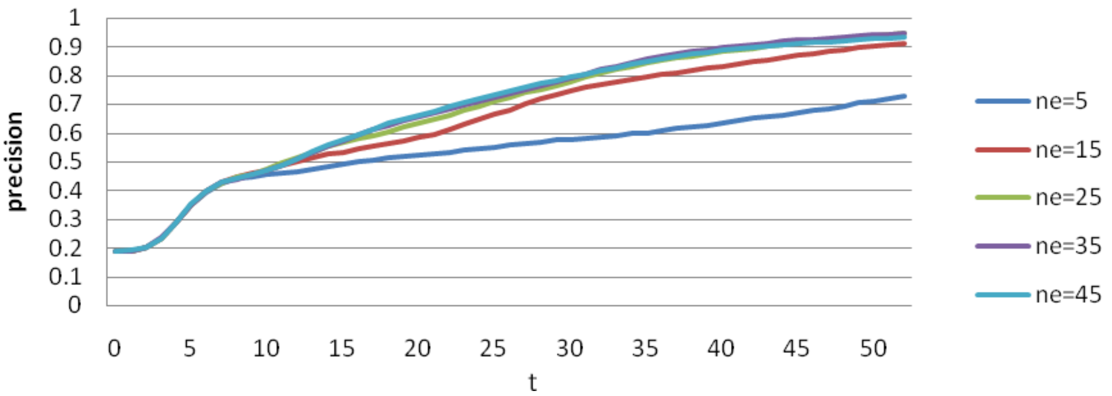


Figure 6 Market environment 2 (average precision calculated for all consumers)



EFFECT OF FREQUENCY OF PURCHASING

Figures 7 through 10 show the time evolution for the average precision in market environments 3 and 4 where we changed the number of neighbors, *ne*, from 5 to 45 in increments of 10. The precision in Figs. 7–9 is calculated for high-frequency purchasers and that in Figs. 8–10 for low-frequency purchasers.

From Figures 7 and 8, we can see that the precision is high at *ne* = 5 in market environment 3. From Figs. 9 and 10, the precision value is high at *ne* = 15 in market environment 4.

Figure 9 Market environment 3 (average precision calculated for high-frequency purchasers)

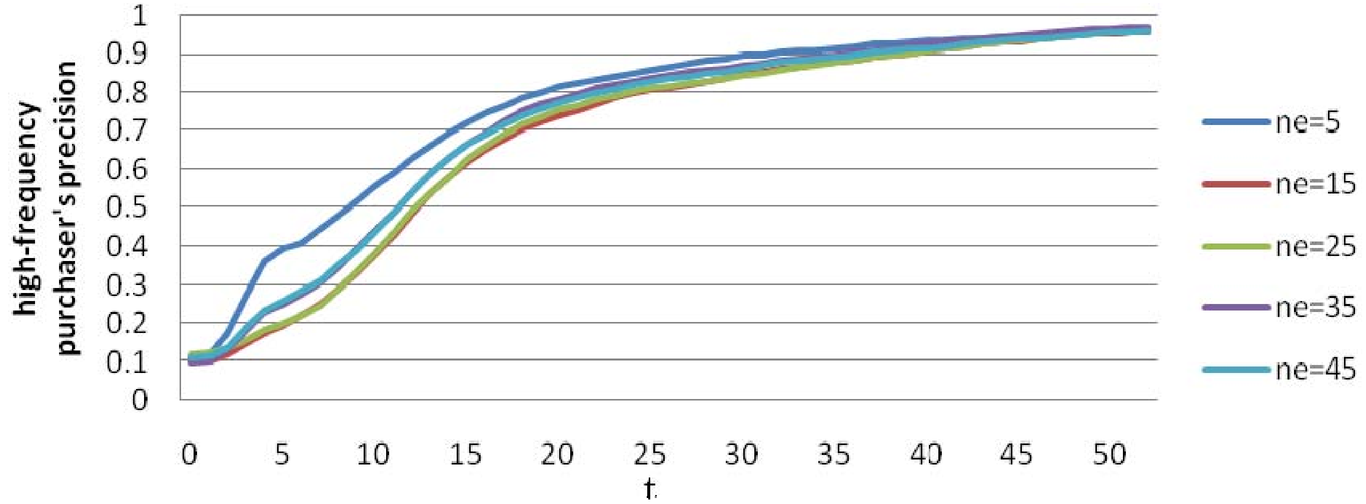


Figure 9 Market environment 3 (average precision calculated for low-frequency purchasers)

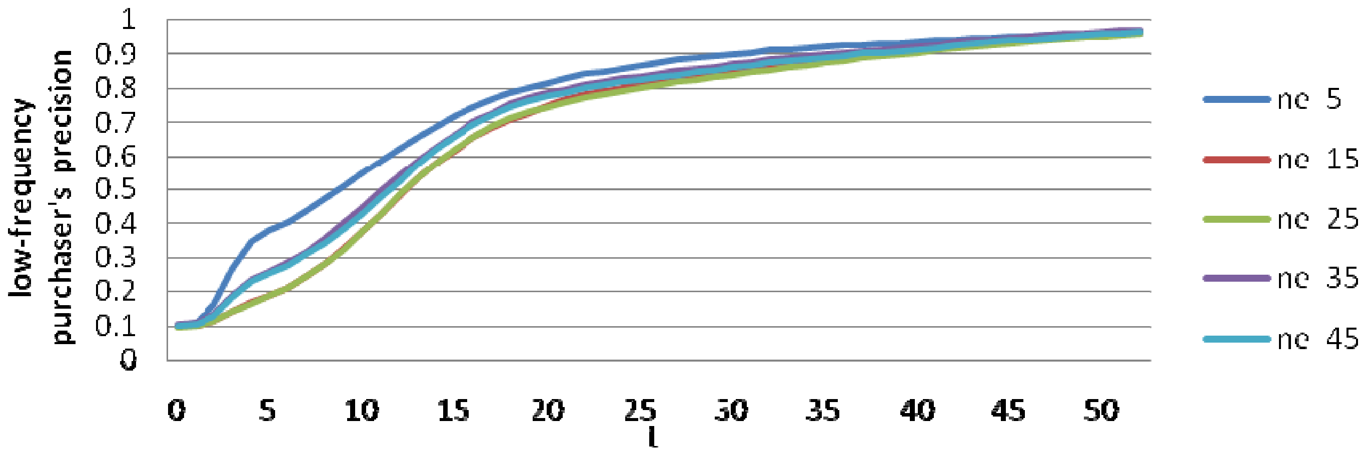


Figure 9 Market environment 4 (average precision calculated for high-frequency purchasers)

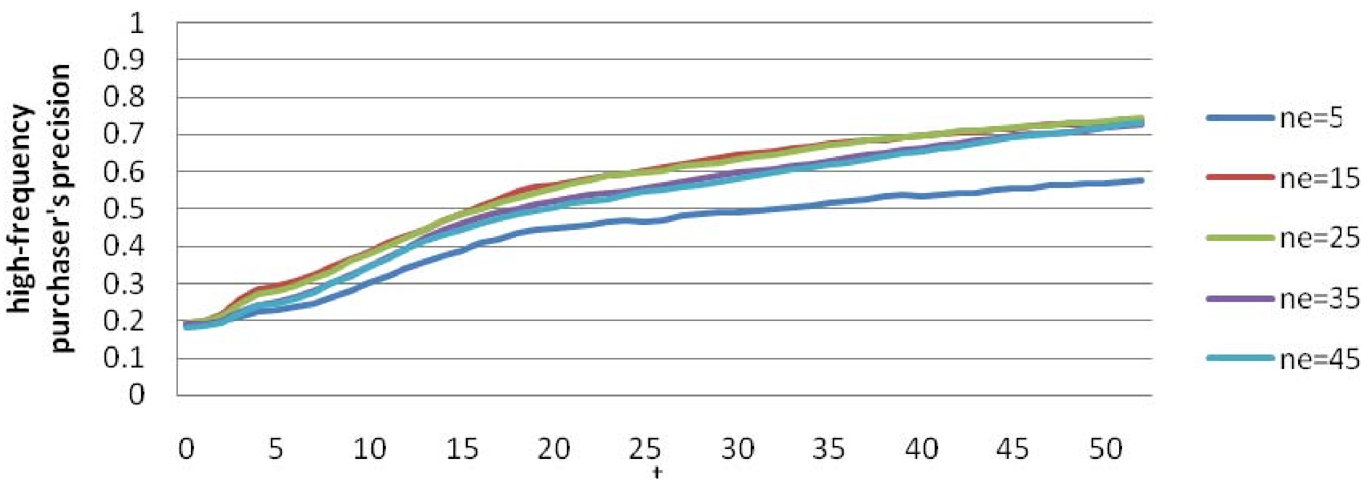


Figure 10 Market environment 4 (average precision calculated for low-frequency purchasers)

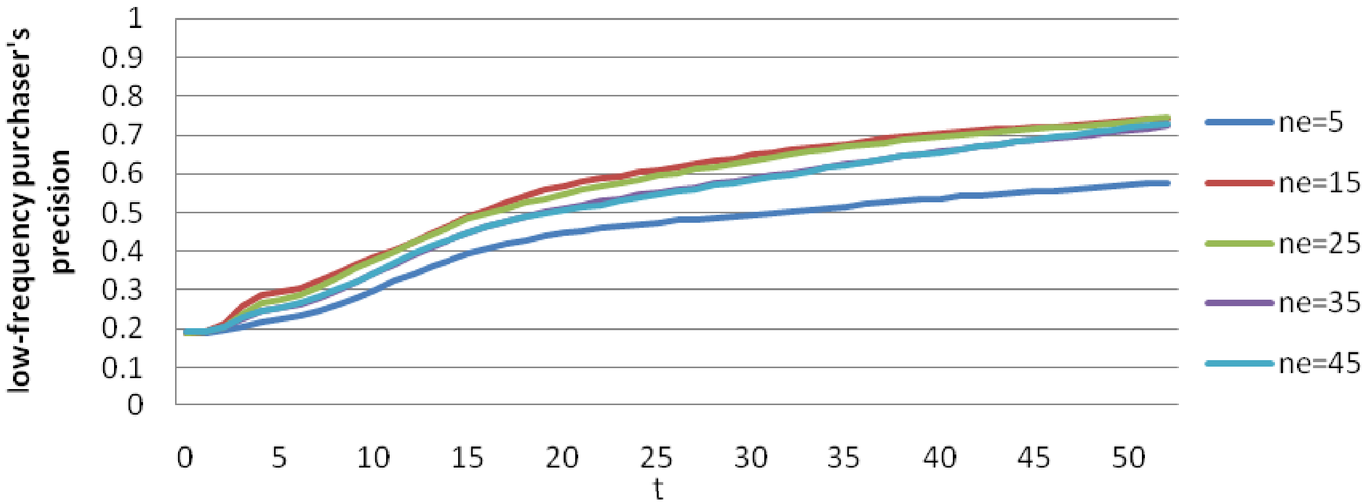


Table 4 summary of results (*ne : optimal value of *ne*)**

		Frequency of purchasing	
Distribution of preferences		homogeneous	heterogeneous
	Simple-cluster	------	$ne^* = 25,35,45$
	Multi-cluster	$ne^* = 5$	$ne^* = 15$

DISCUSSION

From these results, we draw two important conclusions. The first is that the number of neighbors should be bigger in the basic market environment regardless of the clustering of consumer preferences because the collaborative filtering algorithms provide high quality in cases where *ne* is high. We believe that the reason for this is that the algorithms can discover the similar consumers with accuracy. The second conclusion is that if there are any high-frequency purchasers, the number of neighbors should be smaller than that in basic market environment. The reason for this is that a large number of ratings by low-frequency purchasers prevent the recommendation system from finding the neighbors when the number of neighbors is bigger.

CONCLUSIONS

We proposed a new evaluation approach for collaborative filtering, a kind of recommendation algorithm acquired through agent-based simulation. We modeled a virtual E-commerce market where we evaluated the collaborative filtering algorithm. Our findings were as follows: 1) the number of neighbors is a key parameter and there is a trade-off due to market circumstances, 2) a bigger number of neighbors performed better, with a tendency that was independent of the degree of clustering of consumer preferences, and 3) if there were any high-frequency purchasers, a smaller number of neighbors performed better.

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Appendix: The list of variables and parameters in this paper

Expression	Meaning	
IS	A set of items	
CS	A set of categories	
US	A set of consumers	
$sold_{i,t}$	The number of items *i* sold until time *t*	
$BI_{u,t}$	A set of items bought by a consumer *u* until time *t*	
FI_u	A set of items favored by consumer *u*	
$NI_{u,t}$	A set of items that are not bought by consumer *u* at time *t*	
IC_i	The category of the item *i*	
$(pf_{u1},pf_{u2},\ldots pf_{um})$	preference list of consumer *u*	
$Recf(u,t)$	The recommended item for consumer u at time *t*	
$Decf(u,t)$	The item that bought by consumer u at time *t*	
$w1$	The degree of emphasizing on the category of an item	Elements for criteria of decision making whether to buy or not
$w2$	The degree of emphasizing on recommendation from the EC Website	
$w3$	The degree of emphasizing the popularity of an item	
$prob_u$	Purchasing probability of consumer *u*	
$v^t_{u,i}$	The rating value for item *i* by consumer *u* at time *t*	
$ev^t_{u,i}$	The estimated rating value for item *i* by consumer *u* at time *t*	
w_{uj}	The similarity between consumer *u* and *j*	
ne	The number of neighbors	
$NE_{u,ne}$	A set of the neighbors of consumer *u*	
m	The number of categories	
l	The number of items	
n	The number of consumers	