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A PRICE GAME WITH PRODUCT DIFFERENTIATION IN THE CLASSROOM

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ABSTRACT

This paper describes how a small game is used in the classroom to teach theory and decision making. The game is based on a model of product differentiation where firms compete only with prices. Students acting as firms first make decisions sequentially and then simultaneously. Sample results show that the decisions the students make approach the Nash-equilibrium in the game. In a short period of time, the playing of the game shows that the theory can predict price decisions fairly well.

INTRODUCTION

Games are played to give students insights about decisions and decision making in competitive markets (Faria, 2001; Washbush & Gosen, 2001). The complexity in the games reflects the complexity in real markets (Dickinson, 2002; Gold & Pray, 2001; Gold, 2005). That is, games are designed to capture the essence of how reality is related to the purpose of playing them (Feinstein & Cannon, 2002 and 2003). However, greater complexity is not always better (Holt, 1995; Edman, 2005a). In fact, small games can be suitable to teach theory (Shubik, 2001 and 2002). This paper describes a small price game, how the game is played in the classroom in a short period of time, and how it can be used to teach theory and decision making.

The smallest games about competition in markets have only one decision variable (Tirole, 1988). Two of the most used games are the Cournot (Huck, Normann & Oeschler, 2004) and the Bertrand game (Dufwenberg & Gneezy, 2000). Higher quantity than competing firms gives higher profit in the Cournot game. Lower price than competing firms gives higher profit in the Bertrand game. These small games are used for teaching in economics classes (Beckman, 2003; Ortmann, 2003). In games with product differentiation there is also only one decision variable (Garcia Gallego, 1998; Kubler & Muller, 2002; Shubik & Levitan, 1980; Vives, 1999), but the highest quantity or the lowest price does not necessarily give highest profits. Therefore, a game with product differentiation can teach that lowest price and highest quantity do not always give highest profits.

Decisions can be made either simultaneously or sequentially. Most common in games is simultaneous decision making, where all firms make decisions at the same time. Firms can then use information from their own and their competitors' previous decisions, but not their current decisions. In sequential decision making, one firm at a time makes its decision. Firms making decisions later in the decision sequence can take into account decisions already made by competing firms. Firms earlier in the decision making sequence are called price-leaders (Tirole, 1988). Sequential decision making resembles decision making in real markets where not all firms have to make price decisions at the same time.

When playing a game in the classroom, sequential decision making simplifies the procedure for collecting decisions compared to simultaneous decision making. Rather than having students writing decisions on forms and collecting them, or using a computer laboratory, students can just call out their decisions in sequential order. With both sequential and simultaneous decision making in a game, the students can learn about timing of decisions.

The optimal decisions in a game can be used to predict decisions when the game is played. For small games, optimal solutions can be determined analytically. The optimal decisions are then described with costs and parameters of demand. Primarily, there are three solutions of interest (Shubik & Levitan, 1980; Tirole, 1988). First, the cooperative solution, where the profit is highest for all firms combined. Second, the non-cooperative solution (Nash-equilibrium; Nash, 1951), where none of the firms can unilaterally alter its decision to improve its profit. Third, the competitive solution where price equals cost and no firms make any profit. Optimal decisions can differ depending on the timing of decisions. When decisions are made sequentially, firms earlier or later in the decision sequence can have an advantage depending on which game they are playing (Kubler & Muller, 2002).

In the gaming literature, Shubik (2002) describes the uses of small games for teaching theory. For more complex games, Sauaia and Kallás (2003) found in a business game that not many competitors were required to foster competition where firms make decisions towards a Nash equilibrium. Edman (2005b) determined the dynamic Nash-equilibrium in another business game and found that the non-cooperative solution decisions predicted decisions when the game was played better than the cooperative and the competitive solution decisions.

This paper describes how a small game model with product differentiation is used to teach theory and decision making. The game model is presented and its optimal decisions are determined. The procedure for playing the game is described. Sample results when the game is played are compared to optimal decisions, and descriptions of decision making are analyzed. The debriefing after the game is played is described. Some extensions of playing the game are suggested and the conclusion sums up the paper.

GAME MODEL

Shubik and Levitan (1980) describe the game model of product differentiation in detail. The model has price, p, as its only decision variable. The parameters a and b are parameters for demand, g for product differentiation and N for the number of firms in the market. The parameters are the same for all firms in the market. The mean price of all firms in the market is denoted p_m. The studied firm is indexed i. The sum of prices of all firms except firm i is denoted $\sum p_{-i}$, where $\sum p_{-i} = N\, p_m - p_i$.
Demand or sold quantity in units, q_i, of the product for firm i is:

$$q_i = 1 / N (a - b (p_i - g (p_i - p_m))) \quad (1)$$

With price p_i and cost c_i for firm i, the profit for firm i is:

$$\textstyle\prod_i = (p_i - c_i)\, q_i \quad (2)$$

The profit with (1) in (2) for firm i is:

$$\textstyle\prod_i = (p_i - c_i)(1/N(a - b (p_i - g (p_i - p_m)))) \quad (3)$$

The best reply decision is the decision that gives a firm its highest profit with respect to the decisions of other firms. The derivative of the profit (3) with respect to price gives the best reply, p^B, for firm i:

$$p_i^{\,B} = ((bg(\textstyle\sum p_{-i} - c)+N(a+bc(1+g)))/(2b(N(1+g)-g)) \quad (4)$$

Figure 1 shows how the best reply price of one firm is related to mean decisions of the other four firms. The parameter values are the same for all firms when the game is played; a=2000, b=1, g=10, c=300, and N=5. Best price for a firm according to (4) is therefore $p^B = (10(\sum p_{-i} - 300) + 26{,}500)/90$. For mean prices of the other four firms lower than \$470, the best reply price is higher than the mean prices of the other firms, and for mean prices of the other four firms higher than \$470, the best price is lower than the mean prices of the other firms. For example, when the mean price of the other four firms is \$400, the best reply is about \$439, and when the mean price of the other four firms is \$500, the best reply is about \$483.

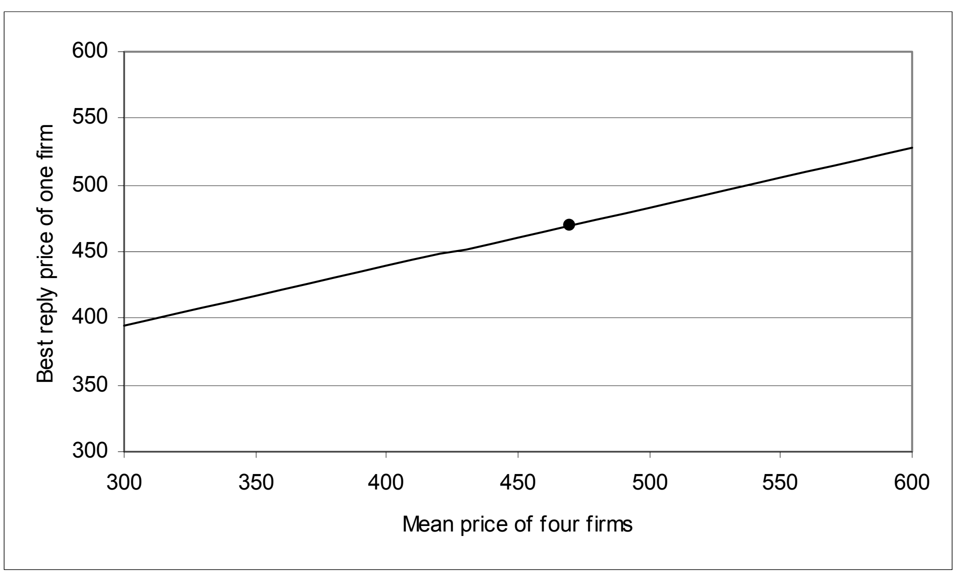
The Nash-equilibrium price, p*, is when all firms make their best reply decisions, p^B. That is, when all firms make the same decisions in equilibrium.

$$p^* = c + N(a - bc) / (bN((2+ g) - g)) \quad (5)$$

The dot in Figure 1 shows the Nash-equilibrium, which is when the best reply price is \$470 and the prices of all other firms also are \$470. Sales are then 306 units and the profit is \$52,020.

When firms make decisions sequentially, the optimal decisions differ from the Nash-equilibrium price. Firms making decisions before the other firms have a higher price. For firms 1 to 5 the following prices, quantities, and profits are in sequential equilibrium: $\{p_1, p_2, p_3, p_4, p_5\}$ = {\$489, \$484, \$480, \$477, \$476}, $\{q_1, q_2, q_3, q_4, q_5\}$ = {287, 298, 306, 313, 315}, and $\{\prod_1, \prod_2, \prod_3, \prod_4, \prod_5\}$ = {\$54,157, \$54,739, \$55,078, \$55,295, \$55,442}. Firms later in the decision making sequence have an advantage and earn

Figure 1: Mean price and best reply price.



somewhat higher profits. The prices are somewhat higher than the Nash-equilibrium price, sales are lower and profits are higher.

With the product substitution parameter g = 0, the prices of the other firms do not have an effect on demand for a firm. The optimal price is then:

$$p^* = c + (a - bc) / (b2) \qquad (6)$$

With the parameters above, the price (6) is $1,150, sales of 170 units and profit $144,500. This price is also the cooperative price, as it is the price that gives all firms the highest profit combined, if they all make the same decisions.

The competitive price is $300. The firms do not make any profits since the price is the same as cost. Firms make the competitive price when product substitution is high ($g \rightarrow \infty$). The non-cooperative price, the equilibrium sequential decision making prices, the cooperative and the competitive prices are used for comparisons to decisions made after the game has been played.

PROCEDURE

The game is played with a spreadsheet, based on formula (1) and (2) above, displayed on the wall in the classroom. The students are ordered into rows with five students in each row (firms 1 – 5). They are informed that they will make decisions for a firm competing with the other four firms in the same row by making decisions on price. The students are told that the cost of the product is $300, and a digital camera is given as an example. The objective for the firms is to earn as much profit as possible.

First, the firms make decisions sequentially (Figure 2). The student acting as firm 1 in the first row is asked to make a price decision. When this decision is made, it is entered into the spreadsheet and displayed so all students can see it. Next, firm 2 in the first row is asked to make its decision, and then firm 3 and so on. Firms 2 – 5 have information about the price decision firm 1 made, firms 3 – 5 have information about the decisions firms 1 – 2 made, and so on.

When all five firms in the first row have made their decisions, the sales of each firm are shown in Figure 2. Then the four graphs in Figure 3 are displayed.

Next, students in the second row make their decisions sequentially and the results are displayed. Usually there are four rows of students making decisions (rows are hereafter called periods). That is, a total of 20 firms play the game. If the number of students exceeds 20, a few of the firms are played by two students. The game is played with sequential decision making for eight periods, which means each firm makes decisions twice. This means, for example, the student playing firm number 1 makes decisions in periods 1 and 5. The second time the firms make decisions (periods 5 – 8) the order is reversed; firm 5 makes its decision first, then firm 4 and so on.

In the following class, the students are asked to make one more decision (hereafter called period 9). The students receive a form with the prices and quantities over eight periods of playing the game, similar to the graphs in Figure 3 and to profits in Figure 4, but with the decisions the students made. The students are asked to study the three graphs and make one more price decision simultaneously for their firms.

The students are also asked on the form "Please describe your thinking when you made your decision" (Blinder, Cancetti, Lebow & Rudd, 1998). These descriptions are collected for two purposes. First, when the participants explain their thinking, they reflect on their decision making. Second, descriptions are used for analysis and are presented at debriefings. The students use about 10 minutes of class time to study the graphs, to type their price decisions and their descriptions. The forms are filled in and returned to the instructor. The results of the decisions are presented at the debriefing.

The game is played as part of a course where bonus points are given to the students toward their grades based on their decisions. In sequential decision making, the students

Figure 2: Computer screen with price decisions and sold quantities.

	Firm 1		Firm 2		Firm 3		Firm 4		Firm 5	
Period	Price	Sales	Price	Sales	Price	Sales	Price	Sales	Price	Sales
1	425	283	385	371	360	426	375	393	500	118
2	400	332	350	442	440	244	450	222	390	354
3	420	267	325	476	435	234	399	313	398	315
4	423	279	395	340	405	318	400	329	400	329
5	415	325	425	303	435	281	440	270	379	404
6	410	374	430	330	445	297	446	295	460	264
7	432	308	457	253	431	311	413	350	414	348
8	429	344	450	298	440	320	450	298	450	298

in each period who made their decisions closest to their best reply decisions are given bonus points. In simultaneous decision making, the eight students who made their decisions closest to the Nash-equilibrium price are given bonus points.

SAMPLE RESULTS

The sample results are from 10 separate class sections, with 20 firms in each section. Over 200 students played the game. Table 1 shows that prices increase and dispersion of prices decreases during the playing of the game. For sequential decision making there are 5 firms, and for

Table 1: Mean prices (standard deviation within parentheses).

Class	Sequential decision making								Simultaneous
section	1	2	3	4	5	6	7	8	9
1	409 (56)	406 (40)	395 (42)	405 (11)	419 (24)	438 (19)	429 (18)	444 (9)	440 (13)
2	364 (15)	354 (59)	375 (14)	387 (17)	551 (253)	451 (58)	443 (47)	467 (41)	451 (26)
3	363 (39)	381 (16)	415 (24)	421 (12)	433 (29)	433 (18)	424 (20)	446 (11)	430 (19)
4	332 (14)	338 (20)	346 (21)	360 (12)	380 (15)	363 (26)	393 (28)	414 (16)	480 (74)
5	383 (44)	352 (20)	390 (8)	388 (14)	410 (16)	434 (7)	444 (6)	444 (3)	426 (21)
6	372 (18)	458 (53)	424 (5)	413 (20)	425 (16)	437 (19)	438 (25)	420 (18)	432 (12)
7	338 (10)	336 (10)	340 (13)	357 (10)	363 (13)	395 (25)	383 (19)	425 (17)	475 (85)
8	351 (29)	412 (81)	383 (12)	384 (10)	397 (34)	545 (258)	397 (41)	500 (81)	423 (38)
9	435 (60)	453 (18)	450 (14)	508 (96)	459 (25)	417 (16)	443 (9)	546 (200)	444 (46)
10	504 (57)	469 (14)	452 (57)	411 (14)	462 (28)	452 (18)	421 (74)	507 (7)	466 (30)

Figure 3: Computer screen with price decisions, sold quantities, profits and price- profit relations.

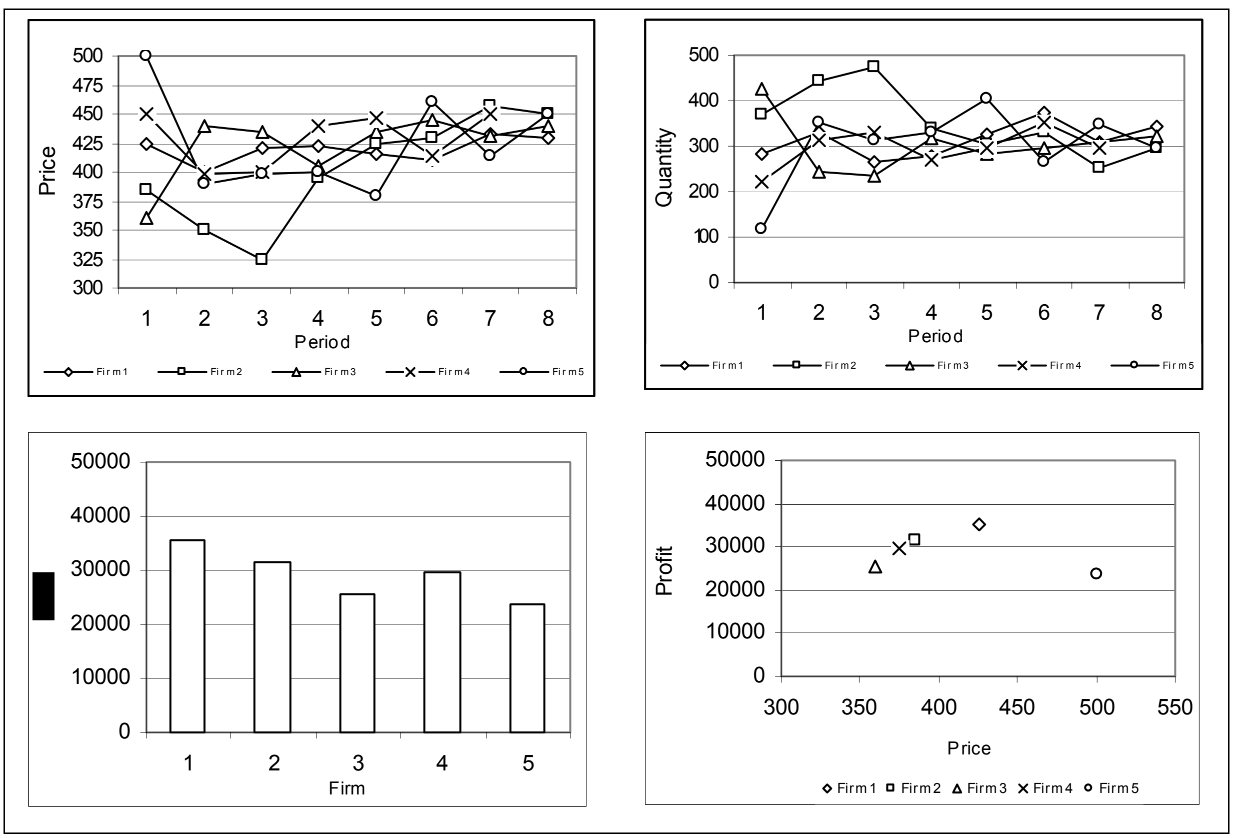
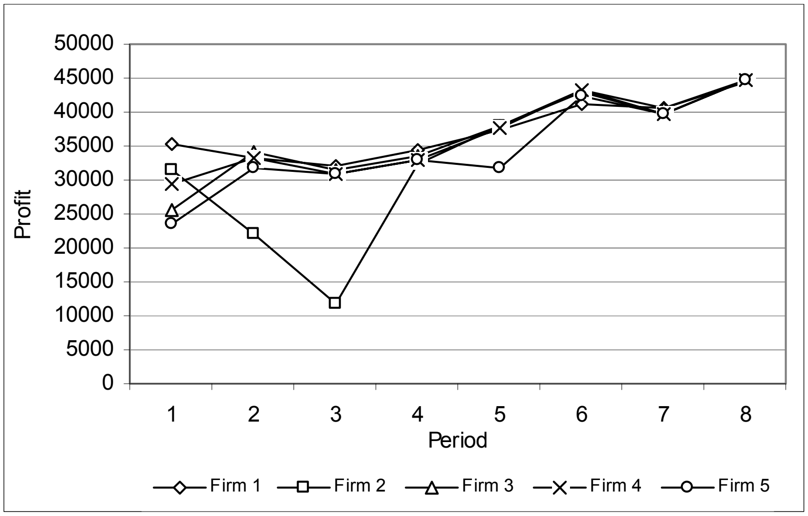


Figure 4: Profits.



simultaneous decision making there are 20 firms. Noteworthy is that prices are still dispersed after nine periods.

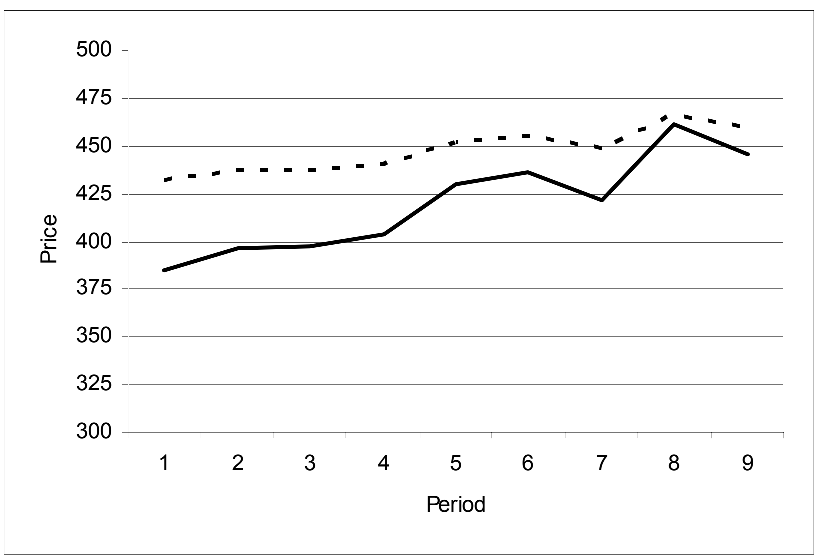
The differences between price decisions and best reply decisions decrease during the nine periods. Figure 5 shows how the mean decisions increase from \$385 in period 1 to \$446 in period 9, and how the mean best reply prices increase from \$432 to \$459. The differences between mean decisions and best reply decisions decrease from \$47 to \$13.

The differences between prices and best reply prices are significant between periods 5 – 8 and 9 ($t(398) = 2.065$, $p = .04$), but not between periods 1 – 4 and 5 – 8 ($t(398) = 1.814$, $p = .07$). The differences between prices and Nash-equilibrium, price \$470, decrease from \$85 to \$24. The differences between prices and the Nash equilibrium price decrease significantly between periods 1 – 4 and 5 – 8 ($t(398) = 4.934$, $p < .01$), and between periods 5 – 8 and 9 ($t(398) = 3.398$, $p < .01$). With decreasing differences between decisions and best reply decisions, the mean profits increase from \$24,225 in period 1 to \$43,238 in period 9.

The sequential decision making does not have any apparent effect on decisions as there were no differences between decisions of firms 1 to 5. That is, firms later in the decision sequence in the same period do not have either lower prices or higher prices than firms earlier in the sequence. Thus, there is no evidence to support the price differences predicted by the theory for sequential decision making.

As mentioned above, the students are asked to describe their simultaneous decision making. Table 2 shows nine

Figure 5. Mean prices and mean best reply prices.



categories of decision making based on the descriptions the participants give on the forms.

The descriptions of decisions correspond fairly well to the decisions made. A majority of the descriptions of decisions (76%) are in three of the nine categories. The description "A price that gave the highest profit" was given for 38% of the decisions. The mean decision made with this description is $450. The descriptions "In the range of earlier prices" and "In the middle or mean of earlier price" are given for 19% of the decisions each. The mean prices in these two categories are $455 and $439, respectively. The mean prices in the three categories were somewhat higher than the mean price $437 for periods 5 – 8 (about 60% of the decisions are in the range $400 and $450, about 20% were below and 20% above).

DEBRIEFING

The formulas (1) to (6), Tables 1 and 2, and Figures 1 to 5 above, are used at the debriefing. The debriefing starts just after the students have made their decisions simultaneously on the forms. The students are asked for their last decisions and these decisions are entered into the spreadsheet and the results are displayed on the wall. The students can see in Figures 3 to 5 that the prices and the profits increase over the periods the game is played.

The Nash-equilibrium concept is presented and formulas (1) to (6) are shown. The students see how the parameters in the game are used to calculate Nash-equilibrium price of $470 and the optimal profit of $52,020. They also get to see how the cooperative price of $1,150 and the profit of $144,500 are calculated. Furthermore, the competitive price of $300 and the zero profit are pointed out. These prices are used for comparison with the prices the students made when they played the game. Table 1 shows evidence that the decisions usually made when the game is played is closer to the Nash-equilibrium price, compared to the cooperative and the competitive prices. The best reply decisions are shown with Figures 1 and 5.

The timing of decision making and the descriptions of decision making is discussed with the students (Figure 2). The minor differences between theoretical decisions in sequential decision making are shown (489, $484, $480, $477, $476) and compared to Nash-equilibrium price $470. Since the sequential decision making does not have any apparent effect on decisions, the theory for sequential decision making does not predict price differences depending on the order in which the firms make decisions (Kubler & Muller, 2002). The reasons may be that the game is not played in sufficient periods and that the price differences in sequential equilibrium are small.

Table 2 is shown to exemplify common descriptions of decision making. One explanation for the decisions made is that the characteristics of the decision making in the game can be described as fast and frugal decision making (Gigerenzer & Goldstein, 1996). In this decision making, good decisions can be made with only little time available and little or no computation of decisions, which contrasts game theoretical assumptions of complete information and rational expectations (Tirole. 1988). The students can compare the descriptions of decision making they made on the forms with the nine categories in Table 2.

EXTENSIONS

The playing of the game can be extended in several ways depending on learning objectives. For a focus on timing of decision making, descriptions of sequential decision making can be captured with forms. Furthermore, based on earlier descriptions of decision making, the forms can have different alternatives of descriptions of decision making which the students can select from. For a focus on computations before making decisions, the time for decision making can be prolonged. For a focus on the number of firms competing in the same market, firms in the same row can be separated into two or more different markets. This may be useful for larger classes. Games with more resemblance to real markets, that is, games with greater complexity, can be used with sequential decision making in the classroom.

Table 2. Description of simultaneous decision making.

Categories of decision making	Relative frequency	Price decisions
A price that gave the highest profit	38%	450 (44)
In the range of earlier prices	19%	455 (50)
In the middle or mean of earlier price	19%	439 (37)
Fair price for consumer and store	8%	424 (43)
Higher prices give higher profits	6%	444 (23)
Price made compared to other prices, undercut	5%	438 (40)
Refer to a specific market	2%	398 (49)
No description given	3%	424 (14)

CONCLUSION

In a short period of time, the playing of the price game with product differentiation shows how theory can predict price decisions fairly well. The sequential decision making makes the game easy to play in the classroom. With both sequential and simultaneous decision making students can learn about the timing of decision making. The game demonstrates that lowest price does not always give highest profits. Descriptions of decision making show that most decisions are simply based on earlier profitable decisions, range, or mean of earlier decisions.

Compared to competition in real markets, the game model is too simple with only price as a decision variable. However, it should be pointed out that the students learn to make price decisions based on information from the game model in which they compete. Furthermore, the simplicity of the game seems to be suitable for the learning objectives.

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