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MONTE CARLO SIMULATION ANALYSIS ON THE COSTS REDUCTION ARGUMENT OF INTEREST RATE SWAPS

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ABSTRACT

This paper develops a Monte Carlo simulation model based on contingent claims theory to evaluate the coupon interest that should be paid by the fixed and floating rate default risky corporate debts. The model improves the Black-Scholes-Merton analysis on firm's liabilities by analysing the coupon paying debts in a variable default-free interest rate environment. The simulation results show that there exist comparative advantage between firms in the fixed and floating rate debt markets, which comes from the firms' balance sheet structure and the nature of business. As such, interest rate swaps can benefit both participating firms for reasons other than market inefficiencies.

Interest rate swaps have evolved as one of the most successful financial innovations in the last two decades. This remarkable development of interest rate swaps has induced great interest of finance researchers to investigate the reasons why interest rate swaps evolved as such a successful financial innovation. One of the most debatable and not yet resolved arguments for the development of interest rate swaps is that interest rate swaps can reduce the borrowing costs of the swap participating firms. Market participants advocate that firms with different level of default risk have a comparative advantage when borrowing in different credit markets. Loeys (1985) and Bicksler and Chen (1986) were amongst the first to point out the existence of quality spread differentials between two firms in the fixed rate and the floating rate markets respectively. When there exists a quality spread differential between two firms, it implies that one firm has a comparative advantage in borrowing fixed rate while the other has a comparative advantage in borrowing floating rate. Two firms can reduce their borrowing costs by borrowing in the market in which they have the comparative advantage and then swap. The short-term floating rate debt is obtained by renewing the short-term borrowings. However, this line of argument has a problem in that the cost savings by the fixed-rate payer in the swap is obtained at the risk of not able to renew the short-term borrowings. Recently, Ungar (1996) showed that a quality spread differential could also exist between a fixed rate coupon bond and a floating rate note. Therefore, the study of how interest rate swaps help to reduce firms' borrowing costs should also be extended to the fixed rate and floating rate coupon paying debt markets. The difference in the borrowing costs between two default risky firms may be reflected in the difference in the coupon payments of the debts issued by the firms. Such difference should be mainly derived from the difference in credit risk between the firms. Finance researchers tend to argue that the reduction in borrowing costs achieved by using interest rate swaps derives mainly from the arbitrage of market imperfections or inefficiencies. Wall and Pringle (1988) is a good survey of the different arguments for the development of interest rate swaps. Subsequent researchers such as Wall (1989), Litzenberger (1992), Titman (1992) and Hull (2000) suggested various market inefficiencies to explain the existence of the quality spread differential. The conjecture is: as the market becomes more perfect or efficient, interest rate swap activities should decline. This conjecture notwithstanding, the rate of growth of the interest rate swap market is still increasing. The arbitrage arguments dominate the literature on the development of interest rate swaps; however, they cannot explain satisfactorily this development given that arbitrage cannot last long when the securities market becomes more efficient. This paper employs the modified Black-Scholes-Merton opting pricing models in the valuation of firms' debts to investigate the quality spread differential issue. The Black-Scholes-Merton option pricing models have been well applied to value corporate debts in order to understand the effects of credit variables on the spread. Merton (1974) first adapted the Black-Scholes option pricing model to the pricing of risky discount bonds. His results yield important insights into the determinants of the credit spread. However, Merton's model suffers from two major shortcomings. First, it considers discount bonds only and second, it assumes fixed risk free interest rate. Neither of the assumptions is prevalent in practice, especially in the case of interest rate swaps. This paper relaxes the assumptions of Black-Scholes-Merton models by analysing the coupon paying debts in a variable default-free interest rate environment. Under these conditions, no closed-form solutions for the valuation of default risky fixed and floating rate debts can be obtained. Instead, I develop a simulation model based on the contingent claims theory to evaluate the coupon interest that should be paid by the fixed and floating rate default risky corporate debts. My model shows that there exist quality spread differentials in the fixed and floating rate debt markets. The source of the quality spread differentials comes mainly from the firms' balance sheet structure and the nature of the business. My results show that interest rate swaps can benefit both participating firms for reasons other than market imperfections or inefficiencies.

The structure of this paper is as follows. Section 2 discusses the problems in the valuation of default risky coupon paying debt securities. Section 3 presents my valuation framework and derives a contingent claims equation for the value of default risky coupon paying debt. Section 4 discusses the simulation model and procedures. Section 5 presents the simulation results and concludes the paper.

THE PROBLEMS IN THE VALUATION OF DEFAULT RISKY DEBTS

The traditional method of valuing debt securities is done by discounting future expected cash flows as specified under the debt indenture at the risk-adjusted rate of return. The risk-adjusted rate of return will incorporate all the risks related to the investment on the debt security. One approach to determine the appropriate discount rate for an individual debt security is to use the Capital Asset Pricing Model (CAPM) to price the risks with the market portfolio. However, the problem of using CAPM is that we inevitably have to use the parameters of expected return and risk preference of investors, which are either unobservable or difficult to evaluate. It is not saying that CAPM cannot be used to value the firm's securities. However, in practice, we still have to use historical values of variable such as β and share prices. Black and Scholes (1973), in their development of the option-pricing model, also recognised that the insights of option pricing theory could be applied to the pricing of corporate liabilities. This is mainly because of the limited liability provision for the shareholders allowed by most corporations nowadays. On debt maturity date or any interest payment date, it can be shown that the corporate equity can be treated as a call option on the assets of the corporation with exercise price of the debt payment needed to be satisfied. Based on the put-call parity, the risk premium of a default risky corporate debt can be treated as a put option with the same parameters as the equity call option that the debtholders have sold to the equityholders. Using the option pricing approach, Merton (1974) carried out the analysis of the default spread between corporate and US Treasury discount securities. His results show that the default spread is a function of a) the level of leverage; b) the volatility of the corporate assets; and, c) the time to maturity of the corporate debt. Merton's (1974) analysis acts as a catalyst for research on corporate liabilities with the application of option pricing or contingent claims theory. However, there are two major problems with Merton's (1974) model. First, Merton (1974) analyses the default spread on pure discount securities but in practice most corporate securities promise coupon payment. In fact, zero coupon securities are relatively rare. Therefore, it is difficult to apply Merton's (1974) model for the analysis of most corporate debts. The difficulty of valuing corporate coupon paying debt is that it will involve a series of options on the assets of the corporation on each debt payment date, each option being dependent on the outcome of the previous option. Second, Merton (1974) assumes the risk free rate is fixed. However, the values of both the default free Treasury and default risky corporate debt are known to be significantly influenced by interest rate risk. Subsequent research has attempted to improve the performance of Merton's (1974) model by dealing with the above two issues. However, the resulting models provide either extremely complicated mathematical solutions or only expressions without closed form solutions.

For example, Geske (1977) first suggested that a series of coupon payments could be treated as compound options in deriving the valuation equation for the coupon-paying bond. However, his results involve a multivariate normal distribution function that is both intimidating and mathematically complex as a consequence of his assumption on the financing of coupon payments. Before discussing the problems with Geske's (1977) model, I would like to point out the important factors affecting the models resulting from using contingent claims technique which attempt to value coupon paying debt securities.

The most important factors affecting the resulting contingent claims models to value default risky coupon paying debt securities are how the event of default or insolvency and the recovery value of the debts in the event of default are defined. It is generally accepted that, in the case of debt securities, default occurs when the borrower is unable to make the contractual payments due on the security at any time during the life of the contract. The assumption of how the contractual payments are financed is important because it affects the balance sheet structure of the borrowing firm and, thus, the derived valuation model. Geske (1977) assumed that the firms finances each coupon payment through a rights issue and states that the firm will find no takers for the stock whenever the value of equity, after the coupon payment, is less than the value of the firm's debts. It is assumed that in this situation the firm is insolvent and default occurs. Based on this assumption, Geske (1977) derived a valuation equation that involves a multivariate distribution function, which is difficult to apply to the study of the risk premium of default risky coupon paying debt securities. The complexity mainly arises from the assumed solvency condition that the value of the firm's assets should exceed the coupon payment as well as the market value of the debt which itself is a contingent claim on the firm's assets.

Longstaff and Schwartz (1995) derived simple closed-form expressions for both risky fixed-rate and floating-rate debt based on a continuous-time option valuation framework. Their model incorporates both default risk and interest rate risk. However, their model explicitly allows for deviations from strict absolute priority and firm insolvency may occur before the contractual coupon payment dates whenever the firm's assets values fall below a pre-defined threshold value. As a result, their model is also mathematically complicated. This paper differs from the papers discussed above by adhering to the more common rule that insolvency can occur only when a firm cannot satisfy the debt service payment due and assuming that the firm finances the debt service payments by selling off its assets. I discuss my valuation framework in the next section.

THE VALUATION FRAMEWORK

I adopt the following notation in developing my valuation framework:

t = number of time periods from 1 to T, with T as the maturity date of the debt;
V_t = the value of the firm's assets at period t;
BF = the face value of the debt;
B_{0T} = the present value of default risk free debt at t = 0 with maturity T;
D_{0T} = the present value of default risky debt at t= 0 with maturity T;
D_t = the payoff of the debt at period t;
XF = the fixed coupons paid at the end of each period t;
XX_t= the variable coupons paid at the end of each period t;
E_t = the payoff of equity at period t;
E_{0T} = the present value of equity at t = 0 with the debt's maturity T;
r_v = the instantaneous expected return of the firm's assets value;
σ_v^2= the instantaneous variance of the return of the firm's assets value;
r = the instantaneous expected return of the default risk free bond;
σ_B^2= the instantaneous variance of the return of the default risk free bond;
RF_t= the default risk free rate at period t.
Z = the standard Wiener process.

The assumptions conventionally made in contingent claims literature are i) there are no transaction costs; ii) Modigliani and Miller's proposition 1 that the value of firm is invariant to its capital structure applies; iii) there are no penalties for short selling; iv) investors can borrow and lend at the same rate of interest; and v) trading in assets takes place continuously in time. In addition to the conventional assumptions, I make specific assumptions on the evolution of default free interest rate and the firm's financial structure and payoff conditions of liabilities in developing my valuation model.

Evolution of Default Free Interest Rate

A1. I allow the default free interest rate to be uncertain and the short-term rate is stochastic. This assumption specifically distinguishes my analysis from previous models, which assume a fixed risk free interest rate. For simplicity, I assume that the default free rate varies over the life of the corporate debt in such a way that the return of a default free discount bond can be expressed as an Ito's process as suggested by Merton (1973):

$$dB / B(T) = r\, dt + \sigma_B (T)\, dz_B(t, T) \tag{1}$$

$\sigma_B(T)$ is assumed to be constant.

The result of this assumption is that the investment in both default free bonds and default risky bonds is subject to the interest rate risk, i.e. the change of market interest rate. This assumption is more appropriate given the actual market environment and enables the model to depict more explicitly the effect of default risk.

Firm's Financial Structure and Payoff Conditions of Firm's Liabilities

A2. The firm has only two classes of claims: a) a fixed rate coupon-paying bond or a floating rate note; and b) the residual claim, equity.

A3. a) The indenture of the bond issue contains the following provisions and restrictions: i) in the case of fixed rate coupon paying bonds, the firm promises to pay a fixed coupon, XF, at the end of each period t and the coupon and face value BF at the bond's maturity date, T; and ii) in the case of floating rate notes, the coupon payment at each period t will be

$$XX_t = BF\ (RF_{t-1} + MK) \tag{2}$$

where RF_{t-1} is the default free rate set one period before the coupon payment date;

MK is the credit risk premium a default risky FRN must pay over a default free FRN and is assumed to be fixed. It is a common market practice of setting periodic interest payments for floating rate notes.

b) in the event of any payment not being met, the bondholders immediately take over the firm and the shareholders receive nothing. However, the firm is limited liability and the shareholders need not compensate for the deficiency of asset value and debt;

c) the firm cannot issue any new senior (or of equivalent rank) claims on the firm nor can it pay cash dividends or do share repurchase throughout the life of the debt;

d) the firm finances the debt payments by selling its assets.

A4. The value of the firm's assets, V, follows the Ito's process:

$$dV / V(T) = r_v\, dt + \sigma_v\, dz_v \tag{3}$$

For simplicity, I assume that there is no correlation between the firm's assets value and the default free rate, i.e. dz_v and dz_B are independent.

Given these assumptions and payoff conditions, the firm's liabilities can be depicted as contingent claims on the firm's assets. For a firm with a fixed coupon bond outstanding, the payoffs of the firm's liabilities will be as follows:

For t = 1 to T – 1;

If $V_t > XF$ $\quad D_t = XF$ (4)

$E_t = V_t - XF$ (5)

the firm is solvent and will go on to another period t + 1.

If $V_t < XF$ $\quad D_t = V_t$ (6)

$E_t = 0$ (7)

the firm is insolvent, dissolved and taken over by the debtholders.

On the debt maturity date, the debt service payment due will be XF + BF. The payoffs of the firm's liabilities will be:

For t = T;

If $V_T > XF + BF$ $\quad D_T = XF + BF$ (8)

$E_T = V_T - (XF + BF)$ (9)

If $V_T < XF + BF$ $\quad D_T = V_T$ (10)

$E_T = 0$ (11)

I start my valuation analysis of fixed rate coupon bonds with a two-period model. The two-period model is used to describe in a simple way the essential theoretical arguments. I show later that my analysis can be extended to a T-period model by a process of induction and the values of fixed coupon rates can be obtained through a simulation model. Given the payoff conditions of debt stated above, the value of debt and equity in the two periods can be written as:

Period 1, t = 1.

$D_1 = MIN(V_1, XF)$ (12)

$E_1 = MAX(V_1 - XF, 0)$ (13)

Period 2, t = 2.

It is important to note that the value of debt and equity in period 2 is based on the condition that the firm is solvent in period 1, i.e. $V_1 > XF$. If not, the firm will be wound up by the bondholders and cease to exist after period 1.

Conditional upon $V_1 > XF$,

$D_2 = MIN(V_2, XF + BF)$ (14)

$E_2 = MAX(V_2 - (XF + BF), 0)$ (15)

Equation (12) to (15) serve as the boundary conditions for the valuation of debt and equity. Equation (13) and (15) appear as two call options with the important difference that the payoff to the period 2 option is conditional upon the payoff to the period 1 option. This is derived from the payoff provisions of the coupon bond.

Based on my assumptions, the expression for the expected value of E_1 can be written in a format that can be generalised subsequently:

$E(E_t) = V_0 \exp(r t) \cdot N(d_t) - D_t \cdot N(h_t)$

for t = 1 (16)

where

$d_t = [\log(V_0 / D_t) + (r + \sigma_V^2 / 2) t] / \sigma_V \sqrt{t}$

for t = 1, 2 (17)

$h_t = d_t - \sigma_V \sqrt{t}$ for t = 1, 2 (18)

The term $N(h_t)$ is of great importance for our presentation since it indicates the conditional probability that $V_t > D_t$. It applies to all the periods subsequent to period 1. In fact, the expected value of equities in periods subsequent to period 1 can also be written in the format of equation (16), but multiplied by the conditional probability $N(h_t)$. Hence, the expected value of equity at period 2 can be written as:

$$E(E_2) = N(h_1)[V_0 \exp(2r) \cdot N(d_2) - D_2 \cdot N(h_2)] \tag{19}$$

Valuation equations can be obtained once we solve the equation for period 1. The equations for subsequent periods are obtained using the same procedure. For period 1, the present value of equity is the present value of the call option. Given my assumptions, the problem becomes the valuation of a call option on the firm's assets under the condition of a stochastic risk-free interest rate. Following the risk neutrality argument of Cox and Ross (1976), we can write the equation of the present value of equity for different periods as:

For 1 period.

$$E_{01} = V_0 \cdot N(d_1) - XF \cdot \exp(-r) \cdot N(h_1) \tag{20}$$

where

$$d_1 = [\log(V_0 / XF) + r + \sigma_v^2 / 2] / \sigma_v \tag{21}$$

$$h_1 = d_1 - \sigma_v \tag{22}$$

For 2 periods.

$$E_{02} = N(h_1)[V_0 \cdot N(d_2) - (BF + XF)\exp(-2r) \cdot N(h_2)] \tag{23}$$

where

$$d_2 = [\log(V_0 / (BF + XF)) + 2r + \sigma_v^2] / \sigma_v \sqrt{2} \tag{24}$$

$$h_2 = d_2 - \sigma_v \sqrt{2} \tag{25}$$

By making use of the balance sheet identity, we can write the present value of the coupon bond, D_{02}, as:

$$\begin{aligned} D_{02} &= (V_0 - E_{01}) + N(h_1)(V_0 - E_{02}) \qquad (26) \\ &= V_0(1 - N(d_1)) - D_1\exp(-r)N(h_1) + N(h_1)[V_0(1 - N(d_2)) - D_2\exp(-2r)N(h_2)] \end{aligned}$$

In order to value a T-period coupon bond, let us define a variable h_0 such that $N(h_0)$ equals one. I create a set of variables defined as follows:

Z_t indicates the conditional probability that the borrowing firm will meet its debt service payments at t, thus, survive into period t + 1. Through a process of induction beginning from equation (26) we can calculate the current value of the T-period coupon bond as follows:

$$D_{0T} = \sum_{t=1}^{T} Z_t (V_0(1 - N(d_t)) - D_t \exp(-r\,t) N(h_t)) \tag{28}$$

$$Z_t = \prod_{0}^{t-1} N(h_t) \tag{27}$$

Equation (28) shows that the key variables affecting the present value of the coupon bond, D_{0T}, are the firm's leverage, the firm's assets volatility, the risk free rate, the face value of the bond and the time to maturity. Comparing two bonds issued by two different firms, the difference in credit spreads should be explained by the firm's leverage and assets volatility.

The valuation logic for floating rate note is the same as above with only XF replaced by XX_t on each interest payment date. The two-period contingent claims model on the valuation of default risky corporate debts provides us important insights that can help us to ameliorate the difficulties of using the CAPM in the valuation of default risky corporate debts. From the above analysis, it is shown that we can use the default free rate to discount the cash flows expected to be received from the corporate debts, which are contingent to the corporate's assets value. With the payoffs conditions pre-defined, we can apply the Monte Carlo simulation method to value the default risky corporate debts. I discuss the simulation model in the next section.

THE SIMULATION MODEL

Fixed Rate Coupon Bond

I can simulate the payoffs of debt on each interest payment date and the debt maturity date based on the value generating processes of the firm's assets and default free rate. I perform a 2-period and a 10-period simulation exercises, i.e. t = 1 to 2 and t = 1 to 10, respectively. Nonetheless, there is no limit in the number of periods used in the simulation exercise. If we choose the annualised figures for the parameters, it implies that the life of the bond is 2 and 10 years. The life of the bond is divided into T steps as $\{0 \equiv t < t + 1 < \ldots < T \equiv 10\}$. This means that $\Delta t = 1$. The value of the firm's assets can then be written in discrete form as:

$$\ln V_t = \ln (V_{t-1} - D_{t-1}) + (r_v - \sigma_v^2 / 2)\,\Delta t + \sigma_v \sqrt{\Delta t}\, z_v \quad (29)$$

where z_v is a standard normal variable. Under the risk-neutral probability principle, we can change the probability measure and write the relevant pricing distribution as:

$$\ln V_t = \ln(V_{t-1} - D_{t-1}) + (r - \sigma_v^2 / 2)\,\Delta t + \sigma_v \sqrt{\Delta t}\, z_v \quad (30)$$

That is, the expected return of the firm's assets, r_v, is replaced by the risk free rate, r, since in a risk neutral world, all returns should be the risk free rate. I allow the risk free rate to be stochastic and, therefore, I have to simulate the risk free rates for each time period. Based on my assumptions and the risk neutrality argument, the value of a default free bond in discrete form can be written as:

$$\ln B_t = \ln B_{t-1} + (r - \sigma_B^2 / 2)\,\Delta t + \sigma_v \sqrt{\Delta t}\, z_B \quad (31)$$

The stochastic default risk free rate, RF_t, for each period is given by

$$RF_{t-1} = \ln (B_t / B_{t-1}) \quad (32)$$

Using a random number generator for the values of z_B and z_v, and determining the payoff conditions of the firm's liabilities as above, I can now generate independent paths for the values of RF_t, V_t, D_t, and E_t for the time periods from 1 to T. By running the simulation 5000 times, the resulting values should approach a normal distribution. Then the sum of the discounted mean values of debt payoffs, D_t, will constitute the present value of the fixed rate coupon bond. Since the debt payoffs can be written as contingent claims on the firm's assets, we can apply the risk neutral pricing argument and use the risk free rate as the discount rate. The present value of the default risky bond should be given by

$$D_{0T} = \sum_{t=1}^{T} \frac{E(D_t)}{\prod_{t=1}^{T}(1 + RF_t)} \quad (33)$$

Instead of calculating D_{0T}, I set D_{0T} = BF and calculate XF so that D_{0T} = BF. The calculated XF of a default risky coupon bond is then compared with that of a default free coupon bond, which should represent the credit spread of a default risky coupon bond.

Floating Rate Note

The simulation exercise for the floating rate note is very similar to that of the fixed rate coupon bond except that the coupon payments of the floating rate note are not fixed but stochastic. According to my assumptions, the floating coupon payments at each period are given by

$$XX_t = BF\,(RF_{t-1} + MK) \quad (34)$$

I then run simulation processes as those described in the fixed rate coupon bond for the values of RF_t, V_t, D_t, and E_t. The present value of the floating rate note at t = 0 is obtained by discounting all the expected values of D_t at the risk free rate. Similarly, by setting D_{0T} = BF, we can obtain the value of MK, which should represent the credit risk premium of the default risky floating rate note. The technical details of the simulation programs for the fixed rate coupon paying bond and the floating rate note can be obtained by writing to the author.

SIMULATION RESULTS AND CONCLUSIONS

The purpose of the simulation exercise carried out in this paper is to study the behaviour of risk premia of fixed coupon bonds and floating rate notes along different times to maturity. The results are expected to provide evidence on the existence of quality spread differentials so that two firms can reduce their borrowing costs through interest rate swaps. I discuss these results below.

Firstly, I check if the results are consistent with the valuation theories of firms' debts. I focus on the behaviour of the fixed coupon rate and the floating rate relative to a firm's leverage and assets volatility, given the default free rate. Table 1 shows a selection of fixed rates values and the mark-up over the default free floating rates. Both the fixed rates and the mark-up increase with the firm's leverage and assets volatility in both short-term and long-term maturity. When the firm's leverage becomes high, e.g. 70% of debt to assets ratio or higher, the fixed rates and the mark-up will increase sharply. Similarly, when the firm's assets volatility becomes high, the fixed rates and the mark-up will increase. However, the impact of an increased leverage on the fixed rates and the mark-up is higher than that of an increased assets volatility. Both the firm's leverage and assets volatility are important factors determining the default risk of the firm. The higher the firm's leverage or assets volatility, the higher the default risk. My results are consistent with debt valuation theories in that a higher fixed coupon rate or mark-up over the default free floating rate for default risky fixed rate bonds or floating rate notes is required when default risk becomes higher.

Secondly, I examine whether or not quality spread differentials exist between two firms with different leverage levels or assets volatility. Tables 2 to 4 show the values of quality spread differentials under three different default free rate volatility environments. I investigate if quality spread differentials exist between the following different groups of firms: a) firms with same assets volatility, but different leverage; b) firms with same leverage, but different assets volatility; and c) firms with different assets volatility and different leverage levels. The results given in tables 2 to 4 show that quality spread differentials are insignificant in an environment with low volatility of the default free rate whereas significant quality spread differentials are found in an environment with high volatility of the default free rate. In an environment of high interest rate volatility, firms are more willing to find ways to hedge against interest rate risk and the existence of quality spread differentials provides a great incentive for firms to choose interest rate swaps amongst other financial instruments. The results given in table 4 also show that quality spread differentials are more significant between firms in groups a) and c) than in group b). This is consistent with the results shown in table 1 in that the impact of leverage is higher than that of assets volatility on the default risk as well as the quality spread differentials between firms.

My valuation model on the default risky fixed and floating borrowing rate sheds light on the source of quality spread differentials. The results show that one of the reasons quality spread differentials exist is the difference in leverage or assets volatility between two firms. The results in this paper reinforce the borrowing costs reduction argument for the development of interest rate swaps by extending the analysis to the more practical situation in which the default free interest rate is variable and the firm's debts are coupon paying. In addition, my model can be extended in a variety of ways. For instance, the model could take into account the different interest rate processes or the influence of different terms of borrowing. Such extension would then provide a theoretical basis for empirical studies on the behaviour of borrowing costs of default risky firms and their interaction with interest rate swaps.

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TABLE 1 REPRESENTATIVE VALUES OF DEFAULT RISKY FIXED AND FLOATING RATES

For $r = 3\%$ $\sigma_B = 3\%$

Time to Maturity T = 2

d^*	σ_v	XF	MK
20	20	3.84	0.03
40	“	3.84	0.04
50	“	3.89	0.08
70	“	4.82	0.96
100	“	47	50
30	10	3.84	0.03
“	20	3.84	0.03
“	30	3.86	0.06
“	50	5.05	1.23

Time to Maturity T = 10

20	20	3.52	0.65
40	“	3.78	0.89
50	“	4.08	0.29
70	“	5.30	1.47
100	“	16.07	13.13
30	10	3.51	0.64
“	20	3.60	0.72
“	30	4.24	0.46
“	50	7.66	3.86

*d = debt / asset ratio in %

TABLE 2 QUALITY SPREAD DIFFERENTIAL BETWEEN DEFAULT RISKY FIRMS UNDER LOW VOLATILITY OF DEFAULT FREE RATE

For $r = 3\%$ $\sigma_B = 1\%$

Time to maturity = 2

Firm	d	σ_v	XF	MK
1	20	20	3.99	0.09
2	50	"	4.03	0.14
3	70	"	4.93	1.03
4	30	10	3.99	0.09
5	"	20	3.99	0.15
6	"	50	5.19	1.29

Time to maturity = 10

Firm	d	σ_v	XF	MK
1	20	20	3.96	0.08
2	50	"	4.54	0.65
3	70	"	5.83	1.94
4	30	10	3.95	0.07
5	"	20	4.04	0.15
6	"	50	8.11	4.23

QSD between firms (in basis points, bp)

a) Firms with same assets volatility but different leverage

	T = 2	T = 10
Firm 1 vs Firm 2	-1	1
Firm 1 vs Firm 3	0	1
Firm 2 vs Firm 3	-1	0

b) Firms with same leverage but different assets volatility

	T = 2	T = 10
Firm 4 vs Firm 5	0	1
Firm 4 vs Firm 6	0	0
Firm 5 vs Firm 6	0	0

c) Firms with different assets volatility and different leverage

	T = 2	T = 10
Firm 1 vs Firm 6	0	0
Firm 3 vs Firm 4	0	1

TABLE 3 QUALITY SPREAD DIFFERENTIAL BETWEEN DEFAULT RISKY FIRMS UNDER MEDIUM VOLATILITY OF DEFAULT FREE RATE

For r = 3% σ_B = 3%

Time to maturity = 2

Firm	d	σ_v	XF	MK
1	20	20	3.84	0.03
2	50	"	3.89	0.12
3	70	"	4.82	0.96
4	30	10	3.84	0.03
5	"	20	3.84	0.17
6	"	50	5.05	1.23

Time to maturity = 10

1	20	20	3.52	0.65
2	50	"	4.08	0.29
3	70	"	5.30	1.47
4	30	10	3.51	0.64
5	"	20	3.60	0.73
6	"	50	7.66	3.86

QSD between firms (in basis points, bp)

a) Firms with same assets volatility but different leverage

	T = 2	T = 10
Firm 1 vs Firm 2	0	2
Firm 1 vs Firm 3	5	6
Firm 2 vs Firm 3	5	4

b) Firms with same leverage but different assets volatility

Firm 4 vs Firm 5	0	0
Firm 4 vs Firm 6	1	3
Firm 5 vs Firm 6	1	3

c) Firms with different assets volatility and different leverage

Firm 1 vs Firm 6	1	3
Firm 3 vs Firm 4	5	6

TABLE 4 QUALITY SPREAD DIFFERENTIAL BETWEEN DEFAULT RISKY FIRMS UNDER HIGH VOLATILITY OF DEFAULT FREE RATE

For $r = 3\%$ $\sigma_B = 5\%$

Time to maturity = 2

Firm	d	σ_v	XF	MK
1	20	20	3.53	0.84
2	50	"	3.59	0.89
3	70	"	4.57	0.85
4	30	10	3.53	0.84
5	"	20	3.53	0.84
6	"	50	4.76	1.13

Time to maturity = 10

1	20	20	2.60	0.87
2	50	"	3.12	0.43
3	70	"	4.20	0.51
4	30	10	2.58	-0.94
5	"	20	2.68	-0.02
6	"	50	6.72	3.09

QSD between firms (in basis points, bp)

a) Firms with same assets volatility but different leverage

	T = 2	T = 10
Firm 1 vs Firm 2	1	6
Firm 1 vs Firm 3	13	16
Firm 2 vs Firm 3	12	10

b) Firms with same leverage but different assets volatility

Firm 4 vs Firm 5	0	2
Firm 4 vs Firm 6	4	11
Firm 5 vs Firm 6	4	9

c) Firms with different assets volatility and different leverage

Firm 1 vs Firm 6	4	10
Firm 3 vs Firm 4	13	17