Volume-Dependent Money Exchange Model for Gaming Simulations

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ABSTRACT

A central issue of international business is the exchange of money, so a money-exchange model is needed in every international-business simulation. Thavikulwat's (1999) volume-independent model rewards pedagogically distracting game playing and requires a disconnected fix to avoid meaningless negative valuation under extreme circumstances. A volume-dependent model is proposed that resolves both problems.

INTRODUCTION

Concurrent with the often-observed globalization of business and the pervasiveness of international-business courses in business-school curricula, college instructors have expressed a need for international-business simulations (Bridwell, 1997). Panel discussions have been presented on international-business simulations (Palia, Yamamura, Cross, Faria, Dickinson, & Roussos,1990; Keys, Edge, & Wolfe, 1992), studies have been conducted on their pedagogical effectiveness (Klein, 1980, 1982), domestically oriented simulations have been adapted to incorporate international content (Halpin & Biggs, 2000), and a scholarly review (Klein, Fleck, & Wolfe, 1993) of international-business simulations is part of the extant literature. Nevertheless, little beyond Thavikulwat's (1995) overall exposition has been published on the algorithmic requirements of international-business simulations.

To be sure, the literature on the algorithmic requirements of business gaming simulations, emerging after Goosen's (1981) seminal publication, is extensive, a recent review of which has been published by Gold and Pray (2001). Thavikulwat's recent work on modeling money exchange rates (Thavikulwat, 1999, 2000) does cover the algorithmic requirements of a central international-business issue, but the volume-independent model he presented is problematic in two respects. First, volume independence permits participants to play with the model such as to detract from the pedagogical purpose of the game. Second, in extreme circumstances and in the absence of a disconnected fix, volume independence can give rise to subsequent money values that are negative, therefore meaningless. Accordingly, if the model's limitation can be avoided by a modification of the algorithm, it should be sensible to do so.

This paper explores the problems of Thavikulwat's volume-independent model and puts forth a modification to the model that allows exchange rates to vary with volume requirements. With the modification, the model gives exchange rates that are the equivalent of dividing each lump sum of money submitted for exchange into infinitesimally small bits, and then exchanging the submitted money for another, bit by bit. As a result, the party demanding the exchange will get the same amount of the other nation's money, irrespective of how it orders the sum to be exchanged, and the subsequent money value can never be negative.

ILLUSTRATION OF THE PROBLEM

Thavikulwat (1999) suggested that the relative value of money should be determined by the ratio of the foreign holdings of each nation's money. Thus,

$$X_{A,B} = \frac{F_A}{F_B}, \qquad (1)$$

where
- $X_{A,B}$: Value of Nation A's money with respect to Nation B's,
- F_A: Foreign holdings of Nation A's money (e.g., $),
- F_B: Foreign holdings of Nation B's money (e.g., £).

Given the model, if a trade transaction requires a trader to exchange one nation's money for another nation's, then as Thavikulwat (1999) explained, the subsequent value of Nation A's money relative to Nation B's will be:

$$Y_{A,B} = \frac{F_A - T}{F_B + U}, \qquad (2)$$

where
- $Y_{A,B}$: Post-exchange value of Nation A's money with respect to Nation B's,
- T: Amount of Nation A's money given to the trader (e.g., $),
- U: Amount of Nation B's money submitted by the trader (e.g., £).

Now, suppose that foreign holdings of Nation A's and Nation B's money amount to A$100 and B£100, respectively, then the volume-independent exchange rate of Nation A's money with respect to Nation B's is equal to their relative value as determined by Equation 1, that is, at A$1 for B£1.

Suppose further that an importer of Nation B wishes to acquire A\$25 for the purpose of paying an exporter of Nation A. Given volume independence, the importer will pay B£25. The transaction, however, changes the post-event money value, and thus the subsequent exchange rate. Applying Equation 2, it will become A\$3 to B£5, because foreign holdings of Nation A's money has been reduced by A\$25, which went to pay the exporter, and foreign holdings of Nation B's money has been increased by B\$25, which was received from the importer. That is,

$$Y_{A,B} = \frac{100 - 25}{100 + 25} = \frac{3}{5}. \tag{3}$$

Playing with the model becomes rewarding when the importer wishes to exchange another B£25 for a second import transaction. This time, the importer will receive only A\$15 (i.e., three-fifths of B£25) in exchange, because of the new money exchange rate. If the importer had combined both exchanges into a single exchange of B£50 at the start, the importer would have received A\$50, which is A\$10 more. Thus, volume-independence biases the terms of exchange in favor of those who are able to aggregate many small transactions into a single large transaction.

As for the second problem, the subsequent relative value of money becomes negative when the amount the importer submits for exchange exceeds the foreign holdings of that currency. This occurs because, with volume-independence,

$$T = X_{A,B}U = \left(\frac{F_A}{F_B}\right)U, \tag{4}$$

which when incorporating into Equation 2 gives

$$Y_{A,B} = \frac{F_A(F_B - U)}{F_B(F_B + U)}. \tag{5}$$

Thus, when $U > F_B$, $Y_{A,B}$ becomes negative.

Of course, the circumstance that can cause the post-exchange money value to turn negative is an extreme one, and the fault can be fixed whenever it would occur by increasing foreign holdings of both nations' money by an equal and sufficiently large amount. Nevertheless, the fix would be disconnected from the basic model itself. It need not be accepted, because a more elegant solution is possible.

SOLUTION

Both difficulties can be resolved by mathematically fragmenting every transaction such that any amount submitted for exchange is exchanged in infinitesimally small bits integrated over the entire sum, so an importer wishing to sell one nation's money for another will get the same deal irrespective of whether the amount sold is in one lump sum or several smaller amounts. This approach requires integral calculus, as follows:

Let du represent an infinitesimally small increment of the amount of Nation B's money that is submitted for exchange, let dt represent the infinitesimally small increment of the amount of Nation A's money that is received in exchange, and let $X'_{A,B}$ be the volume-dependent exchange rate that is desired. Thus,

$$dt = X'_{A,B} \, du. \tag{6}$$

If the exchange rate is to be volume-dependent, then consistent with the logic of the foreign-holdings model for the importing-exporting case,

$$X'_{A,B} = \frac{F_A - t}{F_B + u}, \tag{7}$$

where
  t: the cumulative amount of Nation A's money received in exchange (e.g., \$),
  u: the cumulative amount of Nation B's money submitted for exchange (e.g., £).

Incorporating Equation 7 into Equation 6, we have

$$dt = \frac{F_A - t}{F_B + u} \, du, \tag{8}$$

which can be rearranged as follows:

$$\frac{dt}{F_A - t} = \frac{du}{F_B + u}. \tag{9}$$

If, as before, U represents the entire sum submitted for exchange and T represents the entire sum received in exchange, then

$$U = \int_{u=0}^{u=U} du, \tag{10}$$

$$T = \int_{t=0}^{t=T} dt , \qquad (11)$$

and from Equation 9,

$$\int_{t=0}^{t=T} \frac{dt}{F_A - t} = \int_{u=0}^{u=U} \frac{du}{F_B + u} , \qquad (12)$$

which becomes

$$\Big|_{t=0}^{t=T} - \ln(F_A - t) = \Big|_{u=0}^{u=U} \ln(F_B + u), \qquad (13)$$

and reduces to

$$- \ln\left(\frac{F_A - T}{F_A}\right) = \ln\left(\frac{F_B + U}{F_B}\right). \qquad (14)$$

Thus,

$$\ln\left(\frac{F_A}{F_A - T}\right) = \ln\left(\frac{F_B + U}{F_B}\right), \qquad (15)$$

and

$$\frac{F_A}{F_A - T} = \frac{F_B + U}{F_B} . \qquad (16)$$

After collecting term, Equation 16 becomes

$$T = \frac{F_A U}{F_B + U} . \qquad (17)$$

Thus, the effective exchange rate ($X''_{A,B}$) for an import-export transaction is:

$$X''_{A,B} = \frac{T}{U} = \frac{F_A}{F_B + U} . \qquad (18)$$

For the example problem given earlier, if the importer submits at the start B£50 for money exchange, the importer will receive A$33.33, applying Equation 17 as follows:

$$T = \frac{100 \times 50}{100 + 50} = 33.33 . \qquad (19)$$

If instead, the importer splits the B£50 into two B£25 amounts, the importer will receive A$20 for the first exchange, computed as follows:

$$T_1 = \frac{100 \times 25}{100 + 25} = 20.00 . \qquad (20)$$

For the second exchange, the foreign holdings of both nations' money must first be adjusted for the result of the first exchange, which increased F_B by B£25 and reduced F_A by A$20. So, the importer will receive A$13.33 for the second exchange, computed as follows:

$$T_2 = \frac{(100 - 20) \times 25}{(100 + 25) + 25} = 13.33 . \qquad (21)$$

Thus, for both exchanges, the importer will receive A$33.33, the sum of T_1 and T_2. As the amount received is the same irrespective of whether the importer chooses to exchange the money in one lump sum or two separate amounts, the model does not produce a windfall gain for participants under any circumstance.

This additive characteristic of the model can be proved mathematically for all combinations of U_1 and U_2 that add to U. If a trade-enabling money-exchange transaction of size U is split into two parts, U_1 and U_2, then following Equation 17, the receipt from the first transaction (T_1) is

$$T_1 = \frac{F_A U_1}{F_B + U_1} . \qquad (22)$$

The receipt from the subsequent second transaction (T_2) is

$$T_2 = \frac{(F_A - T_1)U_2}{(F_A + U_1) + U_2} . \qquad (23)$$

Summing both transactions and rearranging terms gives

$$T_1 + T_2 = \frac{F_A U_2}{F_B + U_1 + U_2} + T_1\left(1 - \frac{U_2}{(F_B + U_1) + U_2}\right). \qquad (24)$$

Incorporating Equation 22 into Equation 24 results in

$$T_1 + T_2 = \frac{F_A U_2}{F_B + U_1 + U_2} + \frac{F_A U_1}{F_B + U_1}\left(1 - \frac{U_2}{(F_B + U_1) + U_2}\right) = \frac{F_A(U_1 + U_2)}{F_B + (U_1 + U_2)} . \qquad (25)$$

Thus, because

$$U_1 + U_2 = U , \qquad (26)$$

it follows that

$$T_1 + T_2 = T \,, \qquad (27)$$

proving that the amount received in exchange will be the same irrespective of whether the amount submitted for exchange is submitted in one lump sum or in any two smaller amounts. Generalizing from this result, it follows that the amount received in exchange will be the same irrespective of how the sum may be split for the exchange.

Finally, the post-exchange money value can be gotten by incorporating Equation 17 into Equation 2, giving

$$Y_{A,B} = \frac{F_A - \dfrac{F_A U}{F_B + U}}{F_B + U} = \frac{F_A F_B}{(F_B + U)^2} \,. \qquad (28)$$

Clearly, the post-exchange rate cannot be negative because none of the terms are negative. The post-exchange negative valuation problem is therefore also resolved.

CONCLUSION

The problems of a volume-independent money exchange model are solvable by a modification of the basic model to make it volume dependent, as presented above. Granted, the circumstances giving rise to the problems of game playing and negative valuation may be extreme, but extreme circumstances are not necessarily unlikely when models are used in gaming simulations. Gaming simulations are themselves extreme simplifications of reality. It is not their reality, but their *irreality* that makes them pedagogically valuable.

The real world is cluttered with irrelevant events that detract from learning and confound assessment. Gaming simulations are useful because they reduce the clutter. The world of gaming simulation is a world of very few people and very few nations. It thus is entirely conceivable that one of these very few people might control most of the money that flows among the nations. Models used to exchange money under these circumstances must therefore be especially robust.

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