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INTRODUCING CROSS-ELASTICITIES IN DEMAND ALGORITHMS

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ABSTRACT

Demand for products is determined not only by the usual marketing variables of price, promotion, quality, etc., but also by the interrelationships among those variables. These interrelationships are referred to as cross-elasticities. To better model the actual market place, a simulation demand algorithm that allows not only for changing elasticities, but also for cross-elasticities, needs to be utilized. Building on Teach's Distance Model, this paper describes methodology to incorporate and control cross-elasticities in demand algorithms.

INTRODUCTION

Business literature has long recognized that demand elasticities are not constant over the range of the demand variables and are interrelated (Arora, 1979). Historically, as published in simulation and gaming journals, demand algorithms have generally not included cross-elasticity functions. There is at least one exception, the Executive Game (Hindshaw and Jackson, all editions), that included a rudimentary method to accommodate the cross-elasticity between price and promotion, with no other demand affecting variables included.

Further, in 1984, Teach (1984) introduced a Distance Model that included cross-elasticities, but this concept was not explored in his original papers (Teach, 1984, 1990), nor was a method developed to control them. This paper develops a methodology, based upon the use of the Distance Model, to include the associated cross-elasticities in the demand algorithm. It further demonstrates how the game designer or administrator may control the cross-elasticities and their impact on game outcomes. Only a two variable case is considered, but this work can be extended to the n variable case as well.

BACKGROUND

There are two important concepts in the development of demand algorithms. One concept involves the changes in demand elasticity over the range of the independent demand variables. The Gold and Pray (1983) model is an example of the use of this concept. The second concept is the use of the cross-elasticities of demand variables and their subsequent effects on demand, i.e., the Distance Model. The two models are explained below.

The Gold And Pray Model

At the 1983 ABSEL meeting in Tulsa, Oklahoma, Gold and Pray (1983), introduced a very robust demand algorithm with a modified version subsequently published in Simulation & Gaming (Gold and Pray, 1984). This algorithm, using a set of exponential equations, was unique in that it utilized a methodology that allowed for changing elasticities of demand over the entire range of values for any set of independent variables. However, the model did not allow for interaction effects among the demand variables.

In the Gold and Pray model price elasticity (E_p) is defined as the first derivative of the demand function with respect to price (p) and is shown in Equation 1.

$$E_p = k_1 - [k_2 * p^{(1+\ln(p))}] \qquad (1)$$

K_1 and k_2 are constants that determine the degree of elasticity and are determined by either the game designer or the game administrator to best represent what "reality" the simulation results should represent.

As seen in Equation 1, no variables other than price have any impact upon price elasticity, but having equal price elasticity, when price is held constant, but other demand variables change, is unrealistic. Figures 1 and 2 elucidate the point, i.e., when the amount of the promotional budget is set equal to either $100,000 or $1,000,000, the price elasticities are the same.

Figure 1
Price Demand Schedule
Promotion = $100,000

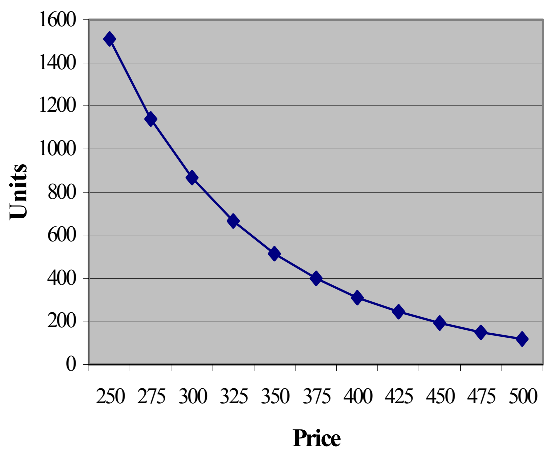
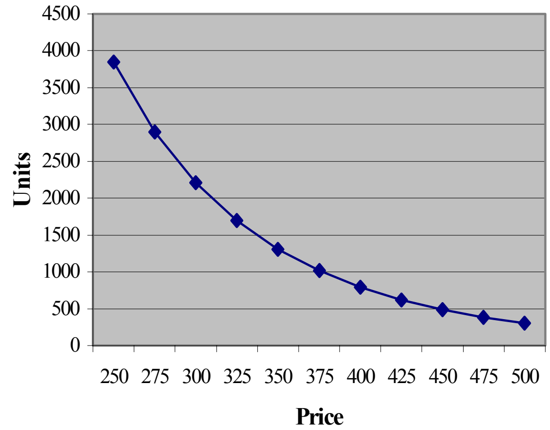


Figure 2
Price Demand Schedule
Promotion = $1,000,000



Thus, the problem with this model is that each of the variables that affect demand are independent or orthogonal to all the others.

The Distance Model

Teach (1984) introduced a "spatial" model to incorporate non-economic variables in the demand algorithm. This model created a “product space” by using product attributes, measured in non-economic units, to determine part of the demand for the products in a simulated market place. The economic variables of price, promotion, quality, and size of the sales force were included in the algorithm by using the Gold and Pray model and combining the results into a hybrid solution.

In the Distance Model, as in most other models, the values used by the equations in the model are not the actual decision values, but instead are exponentially smoothed values. The use of these values prevents large changes in the decision variables from causing wide variations in a simulation's results.

CROSS-ELASTICITIES IN DEMAND ALGORITHMS

In order to include not only the product attribute variables, but also the economic variables in the demand algorithm, the Distance Model needs to be further developed. First, for any economic variable demand calculation to be made, an "ideal" point for each of the variables needs to be located, i.e., differences between the value of the smoothed decision variables and their ideal points have to be determined. The axes' origin needs to be defined in order to perform these calculations and that process is "anchoring the scales." Modifications to the original distances have to be made to account for disproportionate influences of the demand variables. To do this, the scale ranges have to be appropriately modified so that the effect of the demand variables can be controlled by either the game designer or the game administrator. In a similar fashion, the cross-elasticities need to be controlled. To effect this control, a transformation utilizing a non-orthogonal coordinate system is suggested. Finally, based upon the nature of the algorithms design and control, additional modifications may be necessary by the designer or administrator to move the economic "playing field" to the elastic portion of the demand curve. The methodologies are explained below.

The Development of the Ideal Point

In order to develop the ideal point, i.e., the starting point for calculating the allocation of demand across firms, the economic variables must be incorporated into the distance model. In the previous versions of the distance model, only discrete variable product attributes were included (Teach, 1984,1990).

Locating the Ideal Point

It is generally assumed that a lower price is always better than a higher price, but if price is perceived to be too low, there are usually customer suspicions about quality or general distrust of the very low priced producer. As a result, the *Ideal Price Point* for any period is located at the lowest marginal cost point in the industry for that period and if any firm prices above or below this point, that firm's demand is reduced– ceterus parabus. For all other marketing variables, it is assumed that more is better and the ideal point is set at the periods' variable maximums. This is not unlike other models of demand in that higher budgets for advertising, quality, sales force, etc. and lower prices all result in increasing the demand.

By setting the ideal point for each market segment at the maximum of every demand variable, except price, and price set as defined above, a "kink" in the demand curve is created at the current decision points. "Kinked" demand curves are a major economic phenomenon of oligopolies, the type of industries that are most often modeled in business simulations.

Price

A single vector represents the current prices charged by the firms for their products, and firms could have multiple products in the market at the same time. The price ideal point is located at the price that equals the marginal cost of the lowest cost producer. Lower prices are preferred to higher prices, the assumption of all games, except in this model, This model allows for customers to be leery of prices set below marginal cost. Customers prefer the lowest priced product, except if that product was priced below the marginal cost of the lowest cost producer. If a product were offered at a price below marginal cost, customers_would be suspicious and have a tendency to purchase less. (This assumption can easily be altered so that customers always preferred the lowest priced product in the market every period. It is left to the game administrator to define the customers ideal points.)

Promotion

Like price, the promotional budgets for each product in the market can be represented as a vector (assuming that each product has its own promotion budget). If one assumes that promotion is the process that firms use to inform the marketplace about their product and that more promotion is better, the same assumption as all other demand models, then the ideal point would be located at the point on the vector that represents the maximum promotions budget.

Anchoring the Scales

The ideal point for the market segment becomes the anchor point for the scales. For explanation purposes, consider the case where the game has six competitors, each producing a single product and competing in a single market segment. The difference between the ideal price point and the exponentially smoothed price decisions by each firm form a vector with six values. Similarly, the difference between the ideal (maximum) advertising level and each firm's smoothed decisions forms a vector with six vales. The same is true for each of the demand generating variables. Each simulated firm can be represented by an n dimensional point in space, where n equals the number of demand generating variables.

Providing a two variable example might serve to clarify this issue. A firm's exponentially smoothed decisions, the ideal points and the two space coordinate points and the calculations used to determine them are shown in Table 1.

Table 1
Decision Versus Ideal Point Variables

Variable	Firm 1's decisions	Ideal point	Firm 1's two space coordinate points
Price ($)	108	55	53 (109-55)
Promotional budget ($)	250,000	450,000	-200,000 (250,000-450,000)

The Distance From the Ideal Point

The distance between any firm's decisions and the ideal point is calculated by the Pythagorean Theorem. In two dimensional space, the hypotenuse (the distance between two points, $Dist_{Ij}$) is equal to the square root of the square of the difference between the two points on the first axis plus the square of the difference between the two points on the second axis. The distance across n space is shown in Equation 2.

$$Dist_{Ij} = ((a_1-a_2)^2 + (b_1-b_2)^2 + \cdot\cdot(n_1-n_2)^2)^{1/2} \quad (2)$$

Where I defines the ideal point and j defines the firm decisions and j indicates the firm which is being measured (j varies from 1 to the number of competing firms in the industry) and n represents the number of demand affecting variables, both economic and product attribute variables. "a" and "b" are the variables for deriving demand where the maximum number of variables is n.

Defining the Scale Ranges

Using the above equation (2) with the exponentially smoothed decision values would result in the promotion budget, Table 1, dominating the calculated distance, simply because of the magnitude of the numbers. To prevent the largest number in the decision set from dominating the distance, each firm's smoothed decisions are normalized. That is, each smoothed decision, across all firms, is divided by a value serving to normalize the range of all the variables. One such value is the maximum value of each smoothed decision variable. This means that the greatest value for each of the demand generating variables is equal to one.

Weighting the Relative Importance of the Decision Variables

The weighting process allows the game designer or the game administrator to differentially weight the impor-tance of each decision variable. Thus, the weighted distance ($WDist_{ij}$) equation is:

$$WDist_{ij} = (w_1*(a_1-a_2)^2 + w_2*(b_1-b_2)^2 + \cdots w_n*(n_1-n_2)^2)^{1/2} \quad (3)$$

Where w_n is the assigned weight of the nth variable, and the sum of the w's equal n

Demand is allocated to the competing firms inversely proportional to their distances from the ideal point. As the marketing and product attribute variables become closer to the market segment's ideal point, the better the product meets the needs of customers and the greater the firm's market share.

Market Share

The market share for $firm_j$ (MS_j) is defined as the inverse of the distance between $firm_j$ and the ideal point i ($Dist_{ij}$), divided by the sum of the inverses across all n firms as described in Equation 4. Since market share is a quadratic function of distance across all smoothed decision variables greater than zero, then the derivative of the function has all the smoothed decision variables included. Ergo, cross-elasticities for all possible combinations of variables greater than zero exist. A distance of zero simply means that the variable does not contribute to the calculation of demand. Finally, a distance between two points can not be negative.

$MS_j = [(1/Dist_{ij}) / (Sum(1/Dist_{ij})]$ for all ij (4)

Cross-Elasticities

The Pythagorean Theorem is the two-dimensional case of calculating the distance between two points in an orthogonal space using Euclidean measures (taking the square root of a sum of squares based on coordinate points placed on axes set at 90 degrees). The fact that distance is a function of two axes causes an interaction effect and produces the cross-elasticities of the economic variables.

To investigate the cross-elasticity effect, a regression analysis was performed. Distance values (the determinate of demand) were generated using 81 observations. The observations were developed using two variables, each having nine different values in a Latin Square design. The regression forced the intercept through zero. The distance between price and the ideal price as well as the distance between the promotional budgets and the ideal promotional budgets were rescaled to one to nine. The resulting Equation (5) is:

$$Dist_{Ij} = 1.239 * M_j + 1.239 * P_j - 1.057 * [(P_j M_j)^{1/2}] \quad (5)$$

$Dist_{Ij}$ is the distance between the ideal point and the product offered by firm j.
M_j is defined as the distance between the ideal marketing budget and the marketing budget of firm j.
P_j is defined as the distance between the ideal price and the price set by firm j.
PjMj is the cross product distances for firm j.

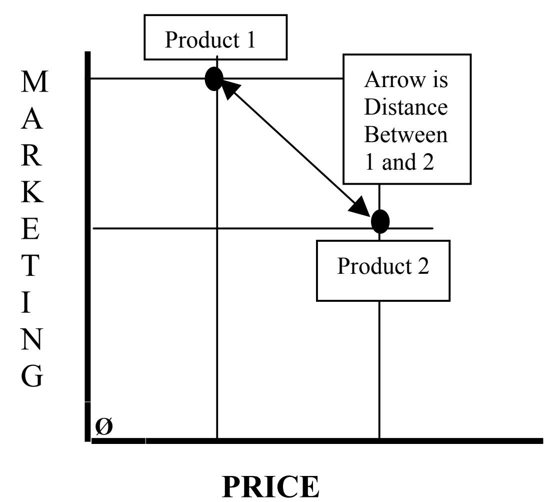
The regression produced a Coefficient of Determination (R^2) of 0.9987.

Thus, Equation 5 accounts for "almost" all of the variance in the distances between the firms' products. One needs to remember that the greater the distance a product is away from the ideal point the less its sales. Thus, the negative value for the cross-product, P_jM_j, means the combination has positive cross-elasticity or greater sales than either price or promotion alone would indicate.

Non-Orthogonal Space

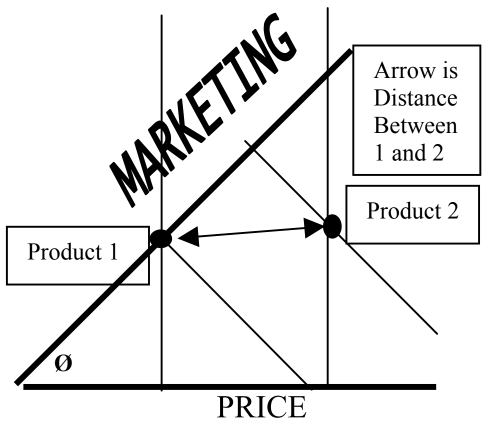
Now that cross-elasticity can be estimated, how can it be controlled for the purpose of game design? The distance calculated in Equation 5 used Euclidean measures. But what if the assumption of 90-degree axes were to be abandoned? That is, a non-orthogonal space, where the angle between the axes may be any angle between 0^0 and 180^0 and not restricted to 90^0. Figure 3 illustrates two products in an orthogonal space.

Figure 3. Two products in Orthogonal Space



If a non-orthogonal space is use, then the length of the straight line between products can be manipulated in a predictable way, and the degree of cross-elasticity can be controlled. Figure 4 illustrates the same problem in a non-orthogonal space.

Figure 4. Two products in Non-Orthogonal Space



Note that the distance between the products has shortened and decreasing the angle *Theta* has reduced the cross-elasticity.

To investigate the cross-elasticity effects relative to a changing angle *Theta,* a series of 11 regressions were performed using the_same 81 data points as before, but with changing values for the angle *Theta* (Table 2). Since the regression coefficients are standardized, they can be directly compared, given an angle *Theta*. Thus, at 15 degrees the cross-elasticity effect is almost as important as the price and promotion variables are, but at 165 degrees the effect is as little as 13 percent. Thus by picking an angle *Theta*, the cross-elasticities can be controlled.

Since the coefficients (beta) are standardized, they can be directly compared, given the angle *Theta*. Thus, at 15 degrees the cross-elasticity effect is almost as important as the price and promotion effects, but at 165 degrees, the cross-elasticity effect is only 13 percent of the price and promotion effects. Thus, by selecting an angle Theta, the cross-elasticities can be controlled.

Table 2
Regression Results with Varying Degrees for *Theta*

Theta	R^2	$beta_1$ Promotion	$beta_2$ Price	$beta_3$ Cross-elasticities
15^0	0.99	2.975	2.975	- 4.803
30^0	0.99	2.298	2.298	- 3.436
45^0	0.98	1.736	1.736	- 2.328
60^0	0.99	1.337	1.337	- 1.551
75^0	0.99	1.064	1.064	- 1.023
90^0	0.99	0.876	0.876	- 0.664
105^0	0.99	0.747	0.747	- 0.417
120^0	0.98	0.658	0.658	- 0.246
135^0	0.99	0.597	0.597	-0.131
150^0	0.99	0.588	0.588	-0.056
165^0	0.99	0.536	0.536	-0.140

THE OVERALL ELASTICITY OF DEMAND

After the distances between the actual and ideal points are calculated, the overall elasticity of demand may need to be adjusted. Since demand is allocated by the inverse of the distances, its elasticity can be altered by raising each distance value to a constant power. If the power is greater than 1.0, the demand elasticity increases and if the power is less than 1.0, the demand elasticity decreases. Table 3 demonstrates this phenomenon for the power of 3.0

Table 3:
Overall Demand Elasticity

Dist	Inverse	Market Share	Power 3 New Distance	Inverse	New Market Share
4.12	0.242	27.9%	70.09	0.0143	46.0%
5.10	0.196	22.6%	132.57	0.0075	24.3%
6.08	0.164	18.9%	225.06	0.0044	14.3%
7.07	0.141	16.3%	353.55	0.0028	9.1%
8.06	0.124	14.3%	524.05	0.0019	6.2%
Sum	0.868			0.0310	

THE DISTANCE MODEL

Although not discussed in this paper, the original model (Teach, 1990) included two important concepts. First, in order to prevent a single product from capturing the entire market, the ideal point has one additional dimension that prevents a perfect match between any existing product and the ideal point. Mathematically this would result in a perfect match that would result in a number divided by zero or infinity. The second concept was that of a *segment shadow*. The shadow absorbed demand, when the products offered in the market were deemed as "less desirable" by the customer. This feature was very important when non-economic aspects such as specific product attributes are included in the model.

SUMMARY

Market share allocations are based upon the relative distance each competing product is from the market segment's ideal point; the greater the distance the less the market share.

All decision variables are exponentially smoothed prior to use in the allocation of demand Equations.

The Ideal-Point for price is set equal to the product with the lowest marginal cost. The Ideal-Point for all other economic variables are set equal to the greatest level of expenditures for each demand-generating variable across all firms.

Each smoothed demand-generating variable is scaled in order that they all have a common range. They are then weighted to represent their desired relative importance to demand.

The angle *Theta* between the axes is determined to set the desired cross-elasticities. If further adjustments to demand are necessary, then the distances between each product and the ideal point are raised by a power function to control the overall elasticity of demand. Finally, the harmonic value of these distances is derived and represents the market share allocated to each product. In equation form, this becomes:

$MS_j = [(1/D'_{Ij}) / Sum(1/D'_{Ij})]$ for all j (6)

Where $D'_{Ij} = (D_{Ij})^P$ for all products in the market place. (7)

Where $D_{Ij} = \{[w_1*(p_j)^2] + [w_2*(m_j)^2] - 2*p_j*m_j*Cos(ø)\}^{1/2}$ (8)

Where:

ø = angle between the axis of p (price) and m (marketing budgets controlling the Price and Marketing cross-elasticities.

m_j= the distance between the highest marketing budget and $firm_j$'s marketing budget, normalized to a common range with price.

p_j = the distance between the lowest marginal cost product and the price set by $firm_j$.

w_1 and w_2 = weights designed to define the relative importance of the demand generating variables

D_{Ij} = The total distance between the Ideal product and $firm_j$'s product

D'_{Ij} = The total distance between the Ideal product and $firm_j$'s product raised to a power to control the demand elasticities as a whole and

MS_j = the market share allocated to $firm_j$'s product

References Available Upon request