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MODELING ATTRIBUTES IN DEMAND FUNCTIONS OF COMPUTERIZED BUSINESS SIMULATIONS: AN EXTENSION OF TEACH'S GRAVITY FLOW ALGORITHM

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INTRODUCTION

The modeling of demand in the marketplace is a critical aspect of any computerized business simulation. Business simulations in particular are supposed to give students insights into the functioning of the "real world." As a result, it is important for the programs to be designed to capture the essential elements and characteristics of the business world.

Teach (1984) found most business simulations made very simple assumptions about the marketplace and the way product attributes were modeled. Most simulations assumed homogeneous products and homogeneous consumer preferences. Marketing variables controlled the allocation of demand including: price, advertising, R & D, sales force, compensation, and channels of distribution. Teach found a few simulations incorporated product attributes directly but only one ideal product was assumed to exist in each market segment; and the market segments were independent of each other.

Considering these problems Teach (1985) developed a demand model allowing for heterogeneous consumer preferences and products with multiple market segments. Within each market segment he created a set of "ideal" product attribute mixes. A firm gains demand within any market segment by developing a product with a mix of attributes that is closer to one of the consumer's "ideal" preferences, but may lose some demand by moving further away from the ideal attribute mix of other consumers. The demand *is* also affected by marketing variables such as: price, advertising, promotion, and R&D. Demand is linked to the Gold and Pray (1983) demand model. To succeed, the firm must understand the preference mapping of the consumers within each market segment, and their relative importance to standard marketing variables.

The attribute model is a significant contribution to demand modeling. The purpose of this paper is to further evaluate and extend the attribute model developed by Teach and link it in a more direct way to the Gold and Pray demand function. The paper proceeds by (1) reviewing the Teach (1985) attribute model; (2) identifying important strengths and weaknesses; (3) developing a revised attribute model with direct links to the Gold-Pray function; and (4) illustrating the revised demand system with a set of examples, evaluating its sensitivity to changes in attributes and elasticities. The importance of our findings and areas of future research are identified.

REVIEW OF TILE TEACH ATTRIBUTE MODEL

The model assumes a simulation of an industry with three firms (1 = 1,2, or 3), each producing a product (P_i) but with different mixes of two attributes (A_1 and A_2) There are three different market segments (S_1, S_2, and S_3) with different preferences but consumers are willing to purchase any product. Market segment 1 prefers only small amounts of attributes 1 and 2. Market segment 2 prefers a large amount of attribute 1 but only a small amount of attribute 2. Market segment 3 prefers a large amount of attribute 2 and a mid-level amount of attribute 1.

Table 1 quantifies the "ideal" levels of product attributes in each segment (S_i). Table 2 quantifies the actual levels of attributes 1 and 2 by firm (P_i).

TABLE 1
Ideal Product Attribute Levels by Segment

	Ideal Attributes	
Segment	1	2
1	1.00	1.75
2	4.00	0.50
3	2.50	3.50

TABLE 2
Product Attribute Levels by Firm

	Attributes	
Firm	A1	A2
1	3.00	1.50
2	2.00	3.00
3	3.75	5.00

The distance each firm (1) is from the ideal mix in each segment (j) is calculated with the formula and the results given in Table 3:

$$D_{ij} = [(P_iA_1-S_jA_1)^2 + (P_iA_2-S_jA_2)^2]^{0.5} \qquad (1)$$

where:

D_{ij} = Distance between Product i and Segment j.
P_iA_1 = Product i level of attribute 1
S_jA_1 = Segment j *ideal* level of attribute 1
P_iA_2 = Product i level of attribute 2
S_jA_2 = Segment j *ideal* level of attribute 2

The inverse of the distance of the firm, relative to the total distance of all arms in the market, determines the market share. This model is similar to the gravity flow model font physical science.

In Table 3 the highest market share (46%) in segment 1 goes to grin 2 (P_2) with the smallest distance (1.60); and likewise for all other segments The market share for firm I in segment 1 (46%) is calculated as the inverse of it's distance 1.60 divided by the sum of the inverses of all firms (0.50 + 0.63 + 0.24 1.37).

TABLE 3
Distances and Market Shares by Segment and Firm

	Distance			Market Share %		
Segment	P_1	P_2	P_3	P_1	P_2	P_3
1	2.02	1.60	4.26	37	46	17
2	1.41	3.20	4.51	57	25	18
3	2.06	0.71	1.94	20	59	21

Shadow Products

Teach also discusses the concept of a shadow product and unmet needs or unfilled market niches. Showing a three-dimensional space configuration of market products and segments, he argues "each of the shadow product's locations will have the same co-ordinates as the market segment, but N units away, on an orthogonal axis, from the market segment. Teach gives an example arbitrarily selecting an "N" of 2. Without showing the intermediate steps, he calculates the impact of adding the shadow product. The results show (1) total industry sales decline from 3500 to 2196, representing unfilled or unsatisfied demand of 1304 units (2) the closer the shadow products are positioned to the market segments (smaller "Ne), the total unmet need will increase.

Adding Economic Variables To the Attribute Model

Economic variables like price and advertising are <u>added</u> to the attribute model Teach modifies the Gold and Pray (1983) functional form for demand to calculate, a distance factor for each firm's economic variables. Using the gravity flow concept, he calculates a total distance by taking the square root of the sum of distances squared for each economic variable. Market share based on economic variables alone is determined by each firm's distance relative to the total distance.

The final marks share considering both economic variables and attributes is determined in a similar fashion. The distance for the economic variables is the inverse of the market share based just on this criteria. The distance for the attributes is the inverse of the market share based just on this criteria. The total distance for both criteria is the square root of the sum of the distances for each criteria squared.

Teach also discusses the possibility of weighing the importance of the economic variables relative to the attributes. In this case Teach multiplies the distance from the economic variables by W. and the distance from the attributes by W_B such that We is between 0 and 1; and W_f= 1-W_a.

CONCERNS WITH TEACH'S MODEL

In the examples, Teach assumes industry level demand is fixed at 3500 units and the distribution of demand *is fixed at* 15% in segment 1, 300/, in segment 2, and 55% in segment 3. This constraint reduces the flexibility and realism of the model. Market segments may not be independent.
His reference to shadow products is not clearly defined. It is not certain whether the shadow product is a new product or just a new attribute. We infer a shadow product contains other attributes, say A3, that are desired by the consumers. In Teach's example it is assumed no firm currently has A). It is not clear if it is possible in his model for a firm to acquire this product and the way in which this would impact his model. Teach only discusses the impact of all firms getting closer or further away from the shadow product and its impact on unsatisfied demand. If attribute 3 is included in the products of some firm but not others, it seems likely total market demand will increase but relative market shares of each of the firms in the three market segments will change.

Teach mentions making the market place dynamic by changing the market segments, preferred product, or attribute mix. It is stated one could identify the beginning and ending ideal points and move the desired point each period of the simulation in a linear or nonlinear path. These are intriguing possibilities but there is no discussion of how this would be done and the sensitivity of the model to such changes.

The way in which the economic variables are linked to the gravity flow model seems complex. The elasticity of any one economic variable or attribute is not defined owing to the way the model is pieced together. The sensitivity of the model to changes in attributes and economic variables is not tested and the stability of the system is not certain given its structure. More testing is needed to demonstrate the behavior of Teach's gravity flow model.

These concerns have prompted the authors to extend the gravity flow model and link it more directly to the Gold and Pray demand function.

REVISED ATTRIBUTE DEMAND MODEL

The system that we are recommending for modeling demand and allowing for market segmentation based on customer attributes is composed of three parts: (1) Gravity Flow Attributes, (ü)a Market Demand System; and *(iii)* Firm-Level Demand System

Gravity Flow Including Shadow Attributes

The gravity flow model is used to describe choice behavior in a situation where product attributes are both continuous and independent. Adding a shadow attribute (A_3) directly to Teach's distance formula yields:

$$D_{ij} = [(P_iA_1 - S_jA_1)^2 + (P_iA_2 - S_jA_2)^2 + (P_iA_3 - S_jA_3)^2]^{0.5} \quad (1)$$

where:

D_{ij}= Distance between Product i and segment j
P_1A_1 Product i level of attribute I
S_jA_1 = Segment $_J$ ideal level of attribute I

The distances D_{ij} and the average distance for the segment (D_j) will serve as variable inputs in the Market Demand System.

$$D_j = \frac{\sum_{i=1}^{n} d_{ij}}{n} \tag{2}$$

The Market Demand System

The full market demand system developed by Gold and Pray involves 10 equations and is described in detail with examples in Gentry [19901. For purposes of this paper only the modifications to the market demand equation and the relevant elasticity calculations will be discussed:

$$Q_j = g_1 P_j^{-(g2+g3Pj)} M_j^{-(g4+g5Mj)} D_j^{-(g6+g7Dj)} \text{ for } j=1,2,3 \quad (3)$$

where:

Q_j is the market demand for the segment J
P_j is the harmonic average price for the industry in segment j
M_j is the average marketing or advertising expenditure for the industry in segment j
g_k is a demand parameter established via elasticity equations

To determine the parameters the administrator must specify the desired elasticities of each independent demand variable at two levels. The elasticity formulas are as follows:

$$g_2+g_3P_j\ (1+l_nP_j) \quad (4)$$
$$g_4+g_5M_j\ (1+l_nM_j) \quad (5)$$
$$g_6+g_7D_j\ (1+l_nD_j) \quad (6)$$

Selecting two levels for the industry over a reasonable range gives two equations with two unknowns and allows the determination of the system parameters (for k=2,7). The selection of g1 will determine the initial market size for that segment. Unlike the constant segment proportion assumed by Teach, relative market segment size will be influenced by the economic variables, average distance, and the elasticities established apriori.

The Firm-Level Demand System

Instead of using the Teach approach, which utilizes the inverse of the distance and then normalizes them to determine firm-level demand, we have opted for the firm-level model described in Gold and Pray (1984). It is composed of a weighing function that uses both economic and attribute distance variables.

$$w_{ij} = k_0 (p_{ij})^{-(k1+k2pij)} (m_{ij})^{-(k3+k4mij)} (d_{ij})^{-(k5+k6dij)} \quad (7)$$

where:

w_{ij} = weight for firm i segment j
p_{ij} = exponentially smoothed price for firm I segment j
m_{ij} = exponentially smoothed marketing $ firm i segment j
d_{ij} = distance from the ideal for firm I segment j
k_0 = scaling factor

The values assigned to the parameters k1,k2,k3..k6 depend on the designer's specification concerning the firm level elasticities. Equation’s (8-10) below are used to determine the values and are solved in the same manner as equations (4-6).

$$K1+K2P_{ij}\ (1+\ln P_{ij}) \quad (8)$$
$$K3+K4M_{ij}\ (1+\ln M_{ij}) \quad (9)$$
$$K5+K6O_{ij}\ (1+\ln D_{ij}) \quad (10)$$

where:

E_{pj} = firm price elasticity in segment j

E_{mj}=firm advertising elasticity in segment j

E_{dj}, firm attribute distance elasticity in segment j

The share equation and firm-level demand calculations are given in equations (11 and 12).

$$S_{ij} = w_{ij} / \sum_{i=1}^{n} w_{ij} \quad (11)$$

$$q_{ij} = S_{ij} * Q_j \quad (12)$$

where:

S_{ij} = share for firm i segment j

q_{ij} = firm demand segment j

Q_j = total demand (all firms) segment j

Total industry demand (Q) is the sum of all segments

$$Q_t = \sum_{j=1}^{n} Q_j \quad (13)$$

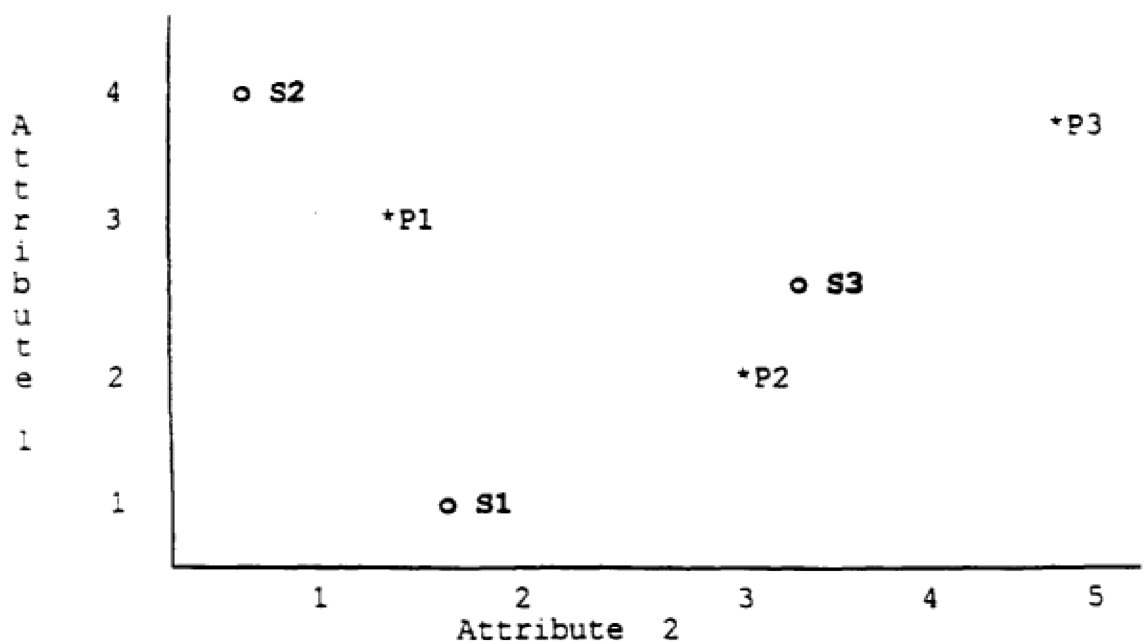
The firms total market share, considering all *three* segments is

$$S_i = \sum_{j=1}^{n} q_{ij} / Q_t \quad (14)$$

ILLUSTRATION OF REVISED ATTRIBUTE DEMAND MODEL

The system will be described and illustrated via a simple example For comparison purposes we will start with Teach's example described previously in Tables 1, 2, and 3. We have 3 firms producing 3 different products competing in the same market place with three different market segments. The product of each firm is characterized by two major attributes that impact the segments.

Graphically it would look as follows:



To determine the market and firm-level demand for each segment, equations 3 - 10 are used for each segment. Price is set up to vary between \$25 and \$35; advertising/marketing expenditures, between \$500 and S 1200. Gravity flow distances based on the scales have their elasticities controlled over the range from 0 to 5.

For this simple demonstration, the elasticities and the starting values are assumed to be identical over each segment. Exponential smoothing is ignored and simple arithmetic averages is employed. The firms starting economic decisions and attribute location values are presented below. The value of the elasticities and parameters are presented below:

TABLE 4
Starting and Ending Values with Associated Elasticities

	Market Level		Firm Level	
Variable	Starting Value/Elasticity	Ending Value/Elasticity	Starting Value/Elasticity	Ending Value/Elasticity
Price	\$ 25 / .95	\$ 35/ 3.00	\$ 25 / 3.5	\$ 35/ 4.50
Marketing	\$500 / .40	\$1200/ 0.15	\$500 / 0.5	\$1200/ 0.08
Distance	0 / .30	5.00 / 1.00	0 / 0.5	5.00/ 3.00

For equation 3, g_1 is preestablished so that industry demand is equal to around 6000 units. The other parameters for both the industry and firm-level demand calculations are presented below:

These starting values and elasticities are used to calculate the industry parameters (g2. .g7) using equations 4 to 6; and the firm parameters (kl..k6) using equations 8 to 10. The resulting market level demand for each segment (j) and weight equations for each firm i n segment j are:

$$Q_j = 0.1712 P_j^{[illegible]} M_j^{+(.5478+4.1E05M_j)} D_j^{-(.2418+.058D_j)}$$

$$W_{ij} = P_{ij}^{-(1.545+.018P_{ij})} M_{ij}^{(.748+6.88e-05M_{ij})} D_{ij}^{-(.292+.207D_{ij})}$$

The Demand Results - Base Case

In tins simple illustration the variability in industry and firm-level demand will be solely based on the gravity flow distances.

TABLE 5
Distances and Average Distance by Segment

	Product 1 (Firm 1)	Product 2 (Firm 2)	Product 3 (Firm 3)	Average Distance
Segment 1	2.02	1.61	4.26	2.63
Segment 2	1.42	3.21	4.51	3.05
Segment 3	2.07	.717	1.96	1.58

TABLE 6
Demand Results By Level: Firm, Segment, Industry

	Firm 1 Demand(Share)	Firm 2 Demand(Share)	Firm 3 Demand(Share)	Segment Demand
Segment 1	761 (.39)	970 (.50)	210 (.11)	1941
Segment 2	1165 (.66)	415 (.23)	194 (.11)	1774
Segment 3	621 (.25)	1218 (.49)	656 (.26)	2495
Demand(share)	2547 (.41)	2602 (.42)	1060 (.17)	6209(1.0)

Interpreting The Results

As would be expected the demand potential for segment 3 is considerably greater than that of the other two segments. In this example this is due to the smaller distance calculation for the attributes. The higher total demand in segment 3 of 2495 (as compared to segments 1 and 2) is due to the firms' coming closer to the ideal in that segment.

Firm 1 gained 66% of the segment demand for segment 2 because its attribute distance value (.42) is much lower than that of firm 2 or firm 3.

Firm 3 has the (east demand potential for both segments 1 and 2 because its distance from the idea! attribute levels is much greater than that of the other firms.

Selecting the "Ideal" Attribute Mix

What would happen if say, Firm I with product I, hit the ideal attribute mix in Segment 1?[1] With the old attributes, Firm 1 only has a 40% share of segment 1, with shares of 66% in segment 2 and 22% in segment 3.

TABLE 7
New Firm Attribute Positions Assuming
Firm 1 Meets The Ideal Mix For Segment 1

ATTRIBUTE	FIRM 1 [P1]	FIRM 2 [P2]	FIRM 3 [P3]
A1	1.00	2.00	3.75
A2	1.75	3.00	5.0

TABLE 8
New Demand Results Assuming
Firm 1 Meets The Ideal Mix For Segment 1

	Firm 1 demand (Share)	Firm 2 Demand (Share)	Firm 3 Demand (Share)	Segment Demand
Segment 1	2099 (.93)	137 (.06)	30 (.01)	2265
Segment 2	622 (.40)	640 (.41)	300 (.19)	1561
Segment 3	554 (.23)	1227 (.50)	661 (.27)	2442
Demand (share)	3275 (.52)	2003 (.32)	990 (.16)	6268 (1.0)

Depending on the elasticities, the firm should capture most of the segment demand if it is the only firm to have the ideal attributes. In this example Firm 1 (P1) increases its market share from 39% to 93%. It also benefits in Segment 3 as its attributes move closer to the ideal in that market.

As would be expected the market grows as the firm comes closer to meeting customer requirements. The absolute amount of the total increase in market demand is controlled by the distance elasticities set at the industry level.

Adding a Shadow Attribute

One of the most important contributions of the gravity flow model is the possibility of adding a "shadow attribute" such as a new technology, or a new customer interest, say, in multicolors of the product.

In a simulation gaming environment, the astute firms would receive customer data during game play on the need for this new attribute. The firm(s) may include this new shadow attribute in their decision scheme and possibly improve their performance in a specified segment.

To demonstrate, let's assume there is a shadow attribute, call it A3, for segment 3 with an ideal value of 5. All firms initially have a value set at 0.

TABLE 9
Ideal Attribute Levels

ATTRIBUTE	SEGMENT 1 [S1]	SEGMENT 2 [S2]	SEGMENT 3 [S3]
A1	1.00	4.00	2.5
A2	1.75	.500	3.5
A3 (SHADOW)	0.00	0.00	5.0

A major strength of this system is that it allows for different elasticities to vary across segments. One Segment may be characterized as price sensitive with low prices; whereas another segment may command a higher price and be more sensitive to product features, service or quality.

Including product attributes allows for a number of significant enrichments to a single-product simulation with standard attributes.

Segment Diversity

By using the system described above in each market segments, significantly different scenarios can be simulated in each segment. One segment can be price elastic; another can be relatively inelastic. Each segment can have different price ranges, different elasticities and different segment sizes. This system allows for multiproduct marketing concepts to be addressed in a single product simulation.

Different segments may be affected by economic and seasonal factors in different ways. One segment may be extremely seasonal; whereas, another may be immune to seasonality. If more than one firm focuses on a small segment, the level of competition prevents a competitive advantage. The beauty of the system is that many different scenarios can be arranged to be consistent with modern economic and strategic management theory. Segments can easily be altered semester to semester, thus eliminating the conventional wisdom that often happens at university-level play.

Changing Market Preferences

Just as process capability requires continuous attention to customer requirements, the game can have moving attributes as the games proceeds. The game administrator can "tighten" or move the ideal locations, thus simulating that customer requirements are changing or getting more difficult to satisfy.

Target Marketing

In most single product games, the economic variables such as advertising, service and product R&D are generally continuous variables and are applied to a single-product industry demand Function. With the segmentation approach, firms can direct a percentage of these economic variables toward a specific attribute and/or segment. Firms would then need to specify the amount and the targeted customer in their decision process.

Total Quality and Customer Satisfaction

Total Quality issues can readily be integrated into the system. If the simulation tracks bad products or failures these can be normalized into an attribute ranging from, say, -3 to + 3. Segments wanting exceptional quality could carry + 3 attribute value.

Customer surveys and other forms of market research can be added, giving teams feedback on how well they are meeting the segment customer requirements - through the attributes and/or the economic variables The customer surveys can also suggest the need for additional attributes.

Simulations have improved dramatically in recent years due to improved demand and cost modeling algorithms. Designing product attributes in the demand function of computerized business simulations will enhance their pedagogical effectiveness and further research is recommended into the scenarios suggested above.

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