Developments In Business Simulation & Experiential Exercises, Volume 19, 1992

INTERACTIVE OPTIMIZATION USING THE METHOD OF RELATIVE IMPROVEMENT PREFERENCES: METHODOLOGY AND EMPIRICAL EVALUATION

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ABSTRACT

This paper presents a revolutionary computerized multiple criteria decision making method. The method was tested in a mathematical programming application by solving a complex multiple criteria production optimization problem. The proposed method, labeled the method of relative improvement preferences (RIP), is compared to the standard "what if" spreadsheet analysis in an empirical examination. Both of these computer-supported methods are appropriate for solving multiple criteria decision-making problems, especially when the number of alternatives is very large. Many interactive techniques are based on the standard what if model. The RIP method proposed in this paper, however, is a departure from the standard model. It is very user friendly, considers competing and non-linear criteria, allows the units of measurement of the various criteria to be different, and operates more efficiently than the spreadsheet model. It uses a scaling algorithm that adjusts for the diversity in impacts caused by problems with heterogeneous criteria and a step-sizing algorithm that rapidly ascends to the optimal solution.

INTRODUCTION

In proposing an improved method for interaction optimization this paper considers a multiple criteria decision making (MCDM) problem. The MCDM problem, which may take the form of a vector optimization program, can be described as follows:

$$\max\ U(f(\mathbf{x})) = V(\mathbf{x}), \qquad (1)$$
$$s.t.\quad \mathbf{x} \in R^m,$$

where x is a subset of Rm, U(•) is a pseudoconcave value function on Rn which is not explicitly known to the user, and

$$f(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_n(\mathbf{x}))$$

is a vector-valued function defined for x = (x1,..., xm) in a compuct convex set x in Rm. Each component $\mathbf{f_j(x)}$, called a criterion function (or objective function), is assumed to be concave.

In essence the DM wishes to choose an (x) (the decision set) from a feasible set (X), which will simultaneously optimize several criteria functions, fj(x). The preferences of the decision maker (DM) constitute a utility function, U(f.(x)), which is not known explicitly. It is assumed that the DM wishes to optimize each criterion's value.

MCDM has been a popular research effort for over a decade. Several methods have been proposed (Larichev & Nikiforov, 1987; Reilly, 1982; Steuer, 1986; Troutt & Hemming, 1984; Wierzbicki, 1980; Yu, 1985) for solving optimization problems that consist of multiple and usually conflicting criteria (an improvement in one criterion's value can be achieved only at the expense of another). The goal is usually to provide a method that will assist the DM in reaching the best compromise solution. Often the criteria functions have different units of measurement (measured on different scales) and therefore it is very difficult for the DM to combine the criteria functions into one overall value function. That is, the non-commensurate criteria can not be combined into a single criterion. The difficulty in solving the multiple criteria problem is compounded further by the vastness in the number of alternatives to be evaluated.

Due to the complexity of the problems to be solved most MCDM methods are interactive computer systems. In "best search direction" methods the DM is asked to provide his or her preference about the criteria values in the form of trade-offs, pairwise comparisons, aspiration levels, rankings, etc. This preference information is used by the MCDM system to find new and better solutions. The procedure is usually one where the solution space is methodically searched for the optimal solution. Several iterations may be necessary and the DM may be required to make numerous agonizing comparisons (Bogetoft & Pruzan, 1991, p.40) and provide preference information at each iteration.

The DM's limitations, in terms of memory and judgement, restrict the amount of information that the DM is able to receive, process, and act on. These limitations necessitate the use of computer supported interactive decision making models, such as those presented here.

Computer spreadsheets and computer search methods are often the only approaches suitable when:

* There are numerous alternatives of input\outputs
* The DM does not know the benefits or possible outcomes related to the choice of inputs.
* The evaluation criteria are conflicting, such as when the DM wishes to reduce costs and increase product quality or service.
* Each criterion is measured in different units, such as dollars, hours, tons, manpower levels.
* The criteria must be dealt with simultaneously (in a holistic manner, rather than the "one alternative at a time" approach).

Furthermore, a computer supported system, such as the one described in sections this paper, is advantageous in that:

* It helps the DM to organize his or her thoughts in terms of alternatives, e.g. desired levels for each criterion's value.
* It rapidly provides values for each criterion.
* It helps identify relationships among the inputs and outcomes.
* It helps reveal the sensitivity of the criteria to different inputs.

THE TRADITIONAL WHAT IF SPREADSHEET (WIS)

A WIS can be used to systematically process the DM's preference of decision variables and step through the solution space. This process, however, is usually time consuming. (A more efficient method is described in the following sections). Computer spreadsheets are especially suited for solving MCDM problems when using the what if approach. Solutions are found in an interactive (iterative) process where the user changes the inputs and observes the new outputs. If output values are not satisfactory the user continues to change the inputs until outputs are satisfactory. There are some disadvantages to using spreadsheets for MCDM. If relationships among criteria and constraints are very complex the user may have difficulty in selecting ideal values for the inputs and some selections may not be feasible, given the constraints. Furthermore, changing inputs in attempts to improve one criterion may cause other criteria to deteriorate drastically. However, using the WIS an experienced DM may rapidly ascend to the best solution, since the user can initially select ideal values for the inputs.

Unfortunately these values are usually not known beforehand. The purpose of the MCDM exercise is to find the ideal values. Newly proposed methods, such as the Relative Improvement Preferences (RIP) proposed in this paper, should at least perform as well as what if spreadsheet models, if not better.

DIRECTION FINDING METHODS

Many of the newly proposed interactive MCDM methods search the solution space for an optimal solution (for a background and discussion of the various methods the reader may refer to Bogetoft and Pruzan, 1991). A fundamental task in these methods is finding the search direction of greatest improvement in the user's utility function, such as the gradient of the utility function. In earlier methods this direction was usually determined from marginal rates of substitution (MRS) among the criteria, e.g. trade-off information. If we assume that the overall objective function, $U(f_1(x))$, can be approximated by the weighted linear function, $w_1f_1(x)+$ then the best direction of improvement of each criterion function, fj(x), at the current solution, x°, is its gradient,

Therefore the best direction of improvement of the linear approximation for U(fj(x)) is the weighted combination of the individual gradients of the criteria, e.g.

$$\nabla f_j(\boldsymbol{x}^0) = \frac{\partial f_j}{\partial x_1^0}, \ldots, \frac{\partial f_j}{\partial x_n^0} \text{ ------ (2)}$$

In the popular GDF method (Geoffrion, Dyer and Feinberg, 1972), the weights, w_1, assessed at some solution point, x°, are

$$\nabla U(f_j(\boldsymbol{x})) = \sum_{j=1}^{k} W_j^0 \nabla f_j(\boldsymbol{x}^0) \text{ ------ (3)}$$

approximated by the MRS between each fj(x) and an arbitrary reference criterion. In the Frank and Wolfe (1956) method, the best direction of improvement is the direction from the current point, x°, to the extreme point, Xe. To find the extreme point the following problem is solved for Xe.

$$Maximize \sum_{j=1}^{k} w_j^0 \nabla f_j(\boldsymbol{x}^0) \boldsymbol{x}^e \text{ ------ (4)}$$

Once the direction is found the next step is to determine how far along the path from x° to Xe to move in order to find the best solution on that path. A common approach to this dilemma is to divide the path into a limited number of equal segments and display the solutions at the division points. (An improved approach is presented in a later section). The user selects the best solution from those displayed and attempts to improve this new solution. This search procedure iterates until no further improvement can be found, or some other stopping rule is used. Similar to the WIS this search process may be very time consuming.

Most search methods have major limitations and consequently suffer a severe reduction in user friendliness (Bogetoft and Pruzan, 1991). Their lack of practicality, in real world problem solving, is perhaps one of their most serious limitations. Other shortcomings are: (1) to find the weights for improving the criteria they utilize complicated trade-off algorithms, (2) they take excessive time to reach a solution, (3) they require confusing exercises, (4) solutions found often diverge before eventually converging on the optimum.

THE METHOD OF RELATIVE IMPROVEMENT PREFERENCES (RIP)

The RIP proposed here is an interactive search method. It is a novel approach to MCDM that provides significant improvements over other interactive methods and is based on the Troutt and Hemming (1984) method of intensities. The DM should not be presented with more information than can be handled at one time. This approach allows the DM to select the best solution from among a limited set of good solutions. Then the DM specifies relative preferences (intensities) for improving the various outcomes (criteria values) of the current best solution. The DM's inputs (preferences relative to the amount of improvement sought) are used to search the solution space for a new solution that is commensurate with those inputs. (Each criterion's weight is simply the DM's preference or desire for improving that criterion's value in relation to the values of the other criteria).

Similar to other MCDM methods the RIP involves the conversion of the MCDM problem, with assumed criteria weights, into an equivalent vector-maximization problem (VMP). An algorithm solves the VMP using the numbers supplied by the DM, for improving each criterion. The central concern of the RIP is to provide as simple a method as possible for soliciting direction of improvement information. It lacks the limitations of ideal point knowledge and is parsimonious with regard to user inputs. The user is not faced with the perplexing task of assigning weights to the various criteria and adjusting those weights as the environment changes. Most MCDM methods require this step. Nor does the RIP present numerous complex decision choices or ranking questions to the user. Instead, it presents good solutions and asks the user to choose the best, and to specify desired improvements in the chosen solution.

When presented with a set of criteria values, f (x), for some alternative solution, **x,** the user applies some number, P, to each criterion, relative to the amount of improvement desired in that criterion's value. This intuitively appealing notion of applying a number relative to the outcome desired is more familiar to the typical user (e.g. the larger the number the more the improvement gained). The RIP assumes that, at any point x, it is desirable to improve each criterion. The computer system then calculates the inputs that are needed (while insuring their feasibility) to yield the desired criteria values (outputs). (Conversely, in the what if method the user changes the old inputs and observes the new outputs achieved.)

The gradient direction is deduced in a manner similar to using the GDF method. A major hypothesis of the RIP method is that these numbers (relative desires for improvement) estimate, up to a positive scaler constant, the gradient direction of U in criterion space. That is

where k is some constant and the P_1 are preferences for improvement.

$$\nabla U(f_j) = kP_j \text{ ---------------- } (5)$$

is a good estimate for

$$\nabla U(f_j) = \partial U/\partial f_j, \text{ ------------ } (6)$$

Since the numerical calculations in the mathematical programming problem of this method are very complex it is vital that the method be computer supported. The complexity is, however, transparent to the user. The user has only to input his/her preferences, e.g. relative desires to improve each criterion's value. The computer quickly provides new solutions for consideration.

THE PROBLEM OF HETEROGENEOUS CRITERIA

Before heterogeneous criteria can be aggregated they must be scaled. Without scaling the RIP is virtually useless because it, like many other MCDM methods, experiences a distortion due to the 'magnitude of impact" problem. For example, if one criterion is to minimize cost, measured in dollars, and another criterion is to minimize workforce fluctuations, measured by the number of workers, there is considerable difference in the magnitudes of the units of measurement. Changes in costs will have a significant impact in the determination of the coefficients of the LP vector, while changes in the number of workers will have little impact. This diversity of impacts gives erroneous results in the linear programming maximization problem.

This is a serious problem since it appears that users have a natural expectation that equal specifications should produce equal benefits. We therefore seek to equalize the impact that each criterion has on the LP coefficient vector that is used to find the extreme point in decision space. To resolve this problem, a scaling algorithm was developed. The procedure provides values for adjusting constants, one for each product (the preference amount times the respective criterion gradient). The application of the preference amount, Pj, by the user will then increase the impact of the respective criterion, relative to the size of the number applied, without undue influence from the criterion's units of measurement.

At any point in decision space the impact that a given criterion, has on the gradient of the value function may be represented by the size of the criterion's gradient (vector). This size may be measured by the norm of the criterion's gradient (vector). To equalize the impacts of each gradient, VCj, at some current solution, x, we find values for the adjusting constants, A_1, such that

$$A_1\|\nabla C_1\| = A_2\|\nabla C_2\| = \ldots A_n\|\nabla C_n\| = 1 \text{ ---- } (7)$$

After scaling the terms in the LP coefficient vector are found by where V is the DM's value function, **xj** are decision variables and C_1 are the criterion functions.

$$\frac{\partial V}{\partial x_i} = \sum_{j=1}^{k} P_j A_j \frac{\partial C_j}{\partial x_i}, \text{ -------------- } (8)$$

Given some starting solution, the Aj for that solution, can be determined. These Aj, along with the user supplied P_1 can be used to find an improved solution. The improved solution will yield still better values of Aj. A most important feature of this method is that as we progress from a good solution to the optimal solution, the values of Aj do not change appreciably from one iteration to the next, and stay constant for linear criteria.

VARIABLE STEP SIZE PROCEDURE

Since the display of an excessive number of alternatives confuses the DM and slows the convergence process, a limited number of alternatives are displayed to the DM. However, identifying which points on the search path yield the best solutions is a perplexing problem. A procedure that uses the sum of the percentage changes (SPC) of the criteria values provides a basis for determining variable interval search lengths. This procedure is based on the assumption that as the DM's compromise solution approaches the optimal solution the amount of change in criteria values diminishes with each iteration. Tables 1 and 2 give example data for three successive iterations, hl, h2, and h3. Notice that the SPC diminishes from a value of 228 to a value of 26.5 as the current solution approaches the optimal.

In our step size determination procedure the ratio of the last iteration's sum of the percentage changes (SPCh-1,) to the current sum of the percentage changes (SPCh-1) is used to determine the variable referred to

TABLE 1
% CHANGES IN CRITERION VALUES (ITERATIONS 1 & 2)

	Criterion Values				
h	C1	C2	C3	C4	C5
1	7500	10	150	20,000	4
2	5000	20	120	15,000	2
change	2500	10	30	5,000	2
%change	33	100	20	25	50
SPC	228				

TABLE 2
% CHANGES IN CRITERION VALUES (ITERATIONS 2 AND 3)

	Criterion Values				
h	C1	C2	C3	C4	C5
2	5000	20	120	15,000	2
3	5500	21	114	14,850	2.2
change	500	1	6	150	.2
%change	10	.5	5	1	10
SPC	26.5				

as p (rho). Let p SPC h,/SPC h. New alternative solutions are thus a function of both p and the extreme point generated. Assume we are using a MCDM method where the old solution (previous best alternative) and four new alternatives are displayed to the DM. The equations for calculating the S_1 (e.g. the three intermediate solutions that lie between the old solution, So, and the extreme point solution, S4) are:

$$S_1 = S_0 + .25(S_4 - S_0)\,\rho \qquad (9)$$

$$S_2 = S_0 + .50(S_4 - S_0)\,\rho \qquad (10)$$

$$S_3 = S_0 + .75(S_4 - S_0)\,\rho \qquad (11)$$

where: p is never > 1 and the S_1 = alternative solutions at the i-th intervals. Whenever the process is not converging p will have a value of 1 and step sizes will be equal. Conversely, whenever the solutions converge, the criterion values converge and p becomes smaller and smaller. Hence, the intervals should decrease so that new alternatives (e.g. intermediate solutions S1, S2, S_3) are closer to the last solution, S0. Relative changes in step sizes (intervals) along the search path for different values of p are demonstrated in figure 1. Note that as p diminishes new alternative solutions are in the neighborhood of the last solution.

FIGURE 1
VARIABLE STEP SIZES AND THEIR RELATION TO ρ.

```
  S0         S1          S2          S3          S4
--*----------*-----------*-----------*-----------*--  ρ=1.0

  S0     S1        S2           S3                S4
--*------*---------*------------*-----------------*--  ρ=0.7

  S0  S1   S2       S3                            S4
--*---*----*--------*-----------------------------*--  ρ=0.2
```

Although at each iteration we search in a new direction, we now search closer to the current solution. Skeptics may argue that this step size determination procedure will result in exclusion from consideration all solutions except local optimum. Note however that although the first step is small the remaining steps are progressively bigger. If the optimum is distant from the current solution (and somehow missed in previous trials) this procedure will still alert the DM to its existence. (At each iteration the solution represented by S_3 is still relatively distant from So).

AN EMPIRICAL TEST

Since there are numerous what-if spreadsheet packages for personal computers (PC), the RIP was tested by comparing its results against those achieved with a WIS. The authors were not able to find the more advanced MCDM models available on PCs, although theoretical descriptions are in the literature.

The test utilized a production planning problem in an artificial decision making environment. Decision variables, x_1, were monthly production amounts of regular time and overtime in an environment of fluctuating demands.

For each of six months in the planning horizon there are two choices that must be made, e.g., the amount of product to be produced by regular time and the amount of product to be produced using overtime labor. A jump in regular time production will cause the hiring of additional workers, whereas a large drop in production will have the opposite effect. Producing in advance and carrying extra product in inventory until needed will increase inventory holding costs and is a trade-off to hiring, overtime, and back order choices. Performance criteria were workforce smoothing index (WFSI), inventory smoothing index (ISI), overtime costs (OT), back order costs (BO), holding costs (HOLD), and hiring/firing costs (HF). These criteria were aggregated to form the value function.

The partials of the value function, V(x), with respect to each decision variable were found, e.g.

$$\frac{\partial V}{\partial x_i} = \sum \frac{\partial C_j}{\partial x_i} = \frac{\partial C_1}{\partial x_i} + \frac{\partial C_2}{\partial x_i} + \,.\,.\,.\,.\,, + \frac{\partial C_5}{\partial x_i} \qquad (12)$$

Next the values of the set of decision variables, x, for the current solution were substituted into the equation for each acj/axi to determine the size (impact) of each criterion's gradient. For each vector (gradient) the mean absolute impact (MAI) was computed.

The MAIs were normalized by dividing by the MAI of one of the criteria, e.g. a base criterion. At one sample iteration the normalized values, (adjusting constants), for the five criteria were 15, 0.004, 1, 1.1, and 0.67 (notice that the third criterion was the base). This wide range in values for the adjusting constants is indicative of the severe disparity in the impacts of heterogeneous criteria.

Tests Design

The research design is shown in figure 2. The convenience sample was four groups of subjects, 10 in each group. Subjects used both the WIS and the RIP, and solved two different production problems.

FIGURE 2
RESEARCH DESIGN

	Sequence of Exercises				
	First Exercise		Second Exercise		Attributes and
Group	Method	Problem	Method	Problem	Outcomes
1	WIS	1	RIP	2	
2	RIP	1	WIS	2	
3	WIS	2	RIP	1	
4	RIP	2	WIS	1	

Each subject participated in two consecutive exercises. To control for order of presentation bias the order was reversed between groups. To control for the learning effect subjects solved a different problem in the second exercise. For example, group one first used the WIS to solve problem one and then used the RIP to solve problem two. The attributes (number of trials and time required) and the outcomes (solutions) were recorded and statistically analyzed.

Findings

Summary statistics for the performance objectives are given in tables 3 and 4 below. Due to paper length limitations data on trials 2, 3, 4, and 5 have been omitted. In addition only averages are shown. An analysis of the data shows that the values of the criteria rapidly converge to an optimal solution when using the RIP, and there is less deviation in

TABLE 3
MEAN VALUES FOR EACH CRITERION, Problem 1

Method	Iteration	WFSI	ISI	OT	BO	HOLD	HF	TOTAL
start-up values		875	700	16000	0	12000	26500	54700
WIS	trial 1	767	965	14723	9623	22660	17933	64939
WIS	trial 6	305	377	7903	885	11992	11966	32746
RIP	trial 1	124	220	8636	0	8480	8783	25899
RIP	trial 6	106	253	1677	36	6396	9668	17776

TABLE 4
MEAN VALUES FOR EACH CRITERION, Problem 2

Method	Iteration	WFSI	ISI	OT	BO	HOLD	HF	TOTAL
start-up values		2333	157	19250	360	26783	79500	125893
WIS	trial 1	1939	2086	18461	7423	68366	41181	135432
WIS	trial 6	341	1144	20450	5455	54002	22741	102649
RIP	trial 1	256	118	24643	275	17839	27213	69970
RIP	trial 6	143	105	26097	0	1424	19180	59401

criteria values from one trial to the next trial. In the WIS values of the performance criteria did not improve during the first two or three iterations, and only improved slowly in later trials. Conversely, using the RIP subjects found the best mix of inputs and criteria values much quicker. (Notice, that in this problem situation, the user is always attempting to minimize the value of each criterion.)

In problem 2 the relationships among the various criteria were somewhat unique in that a good mix of criteria values consists of high levels on a specific criterion, overtime costs, in order to achieve significantly lower levels on several other criteria. Using the WIS, subjects did not recognize this possibility and kept values on that criterion so low that the best mix of values did not occur. In the RIP, however, subjects quickly found the best mix of values.

SUMMARY

The RIP was shown to be far superior to the traditional what if interactive method in the application tested. Although the data is not shown here, both the number of trial solutions and the time required to reach an acceptable solution were significantly reduced when using the RIP. Acceptable solutions were achieved in half the time required by the WIS, yet the RIP gave much better solutions.

At each trial the aggregate value of the cost criteria was better when using the RIP. In addition, the aggregate value of the workforce and inventory smoothing indexes was better. In response to a questionnaire comparing the two methods a two-thirds majority of the subjects said that the RIP was easier to use than the WIS. Two-thirds of the subjects also said, however, that the WIS was more clearly understandable. Subjects did not understand the internal functions of the RIP. In contrast many subjects were familiar with what if spreadsheet packages such as LOTUS 1-2-3. In general, the utilization of RIP was easily grasped by the subjects.

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