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THEORETICAL DERIVATION OF A MARKET DEMAND FUNCTION FOR BUSINESS SIMULATORS

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ABSTRACT

A theoretical derivation of a market demand function is presented. The derivation begins with the principle of utility maximization and yields a function, which conforms to the law of demand, the law of diminishing returns, and the concept of ordinal utility. The derived function should be suitable for a wide variety of applications. The derivation process serves as a proof for the market demand function derived, and it highlights some of the information that ought to be included in published computerized business simulators.

INTRODUCTION

Computerized, competitive, business simulators are powerful pedagogical devices. They allow students to develop their rational decision-making skills while having to cope with subjectivity due to personal values, risk-taking propensity, and the uncertain behavior of competitors. The students get direct and impartial feedback on the financial effectiveness of their decisions, and they can simulate the decision-results cycle many times to improve their skills. However, the value of the learning acquired depends, in some measure, on the degree to which the simulation is realistic (Dittrich, 1977).

Considerable effort has been devoted to improving the design of the market demand functions used in computerized business simulators since Pray and Gold (1982) found reason to question the theoretical and behavioral validity of some functions then in use. Gold and Pray (1983) developed a new function to model market share and market demand, and showed (Gold & Pray, 1984) how the function satisfies the microeconomic principles of the law of demand and diminishing returns. Goosen (1986) provided a graphical approach for incorporating Gold and Pray's (1983,1984)-market demand function in a computerized business simulator.

Decker, LaBarre and Adler (1987) developed two equations to achieve the same result as Gold and Pray, but with two additional features. The functions include an "optimum" value of the demand determinant used in the function. By using a set of reference values, the concept of ordinal utility is incorporated into the simulator. The set of optimum values, one for each demand determinant, can be interpreted as the consumer's point of indifference. The other feature is an administrator-controlled parameter to provide a family of

functions with one set of code, rather than the usual single function. This feature can be used to keep students from "learning how to beat the game".

Teach (1984) provided a geometric model for calculating market share to be used with Gold and Pray's (1983) market demand function. Teach's (1984) model also incorporates the ordinal utility concept. Thavikulwat's (1989) simulator does not model competition for market share, but it does incorporate a reference price concept, and it adjusts period demand for the values of the non-price demand determinants. Furthermore, it incorporates the product life cycle concept to model the long-term trend in demand.

While the work just reviewed constitutes an important advance in the design of business simulators, it is deficient in one important respect: it provides no guidelines for choosing between the alternative constructions of the market demand function. The purpose of this paper is to provide a remedy for this deficiency. The guidelines provided are in the form of a process for deriving a demand function.

The process begins with a description of the type of product to which the derived market demand function applies. Next, the consumer's utility function for the product is obtained. From the nature of the product and the consumer's utility function, the shape of the univariate demand function is derived. Then an exact equation for the univariate demand function is obtained. Finally, the market demand function is obtained as a product of the univariate demand functions.

It should be noted that the process constitutes a theoretical proof for the existence of the demand function derived in this paper. The paper concludes with a discussion of the implications of the process for the design and publication of computerized business simulators.

THEORETICAL DERIVATION

In microeconomic theory, utility, marginal utility, and demand are assumed to be multivariate functions of a set of several variables. This assumption is supported by empirical research. It is assumed herein that these determinants are all mutually independent. With the independence assumption, each multivariate function is the product of all the univariate functions. Also note that the independence assumption is necessary for proper use of the *ceteris Paribus* assumption. In this paper the terms demand function,

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utility function, and marginal utility function will mean the univariate function. Market demand function will mean the multivariate demand function. The word determinant is used rather than predictor because the function to be derived is intended for use in computerized business simulators.

Product Description

In the derivation that follows it will be assumed that the specific product (not a class of products such as medical services) is one that consumers will not hold in inventory; rational consumers will buy one unit and not buy another until the previous purchase has been completely consumed. Products which satisfy the no inventory assumption include a haircut, a vacation package, and a medical exam. The number of products for which the demand function to be developed will apply can be expanded considerably by relaxing the no inventory assumption slightly. For example, if one does not count as inventory an obsolete model of a product that has been replaced but not discarded, the demand function will apply. With the relaxed inventory condition the demand function to be derived will apply to many consumer durables, such as kitchen appliances (both large and small), lawn and garden tools, exercise equipment, and electronic devices. It is also assumed that the product is designed for a particular market segment (Buzzell & Gale, 1987).

Consumer's Utility Function

A fundamental principle in consumer demand theory states that the consumer's marginal utility versus quantity relationship for a product is the consumer's demand function for that product. The product's demand function is the combination of all consumers demand functions. Therefore the first step in deriving a product demand function is to derive the consumers' demand functions.

The key to deriving a consumer's demand function for a product lies in the assumption about inventory at the consumer level. When a consumer holds no inventory of a product, the total utility for that product is equal to the utility of one unit of the product. The consumer's utility and marginal utility functions are a vertical line on the utility versus quantity graph. Therefore, demand is perfectly inelastic for the individual consumer.

As stated above, a product's demand function is the combination of all consumers' demand functions. If inelastic consumer demand functions are combined by simple addition, the result is an inelastic product demand function. Products with inelastic demand are products, which are absolutely necessary for maintenance of life. Since the products assumed in this paper are not in the category of absolute necessity, simple addition is not the proper combination rule.

The proper combination rule is found by applying the principle of utility maximization, using the ceteris paribus condition. According to this principle, for each consumer there is a marginal utility/price ratio, (MU/P) at which the consumer will buy the product assumed. This "indifference point" MU/P value is determined by the bundle of all

products, other than the product assumed, that the consumer is considering buying during the period.

With the no inventory assumption, the quantity purchased in any time period is the proportion of the market segment that will purchase at the product's MU/P value multiplied by size of the market segment. The phrase "proportion of the market segment that will purchase" will hereinafter be shortened to proportion of purchase.

The proportion of purchase is a probability distribution function of MU/P. Therefore, to derive the demand function the proportion of purchase distribution must be known.

Proportion of Purchase Distribution

A product's MU/P value changes by changing price and/or changing the real and perceived attributes of the product. Changes in these demand determinants, however, do not cause changes in the elasticity of demand. The inelastic demand for the product assumed is determined by the no inventory condition.

It is easiest to derive the proportion of purchase distribution by first considering proportion of purchase as a function of price, with all other determinants of demand held constant. As price approaches infinity, MU/P approaches zero. In the limit the product has no relative utility and will not be purchased by anyone. The proportion of purchase will equal zero when price equals infinity, but realistically it will approximate zero at some price considerably less than infinity.

There will be some price less than infinity at which one consumer will buy. At this price the proportion of purchase is $1/N$, where N is the size of the market segment. For this first buyer, demand is perfectly inelastic. Therefore, the demand function must asymptotically approach perfect inelasticity as price approaches infinity.

When the price is slightly less than the price at which one unit is purchased, the MU/P ratio is very low and the price is high. The proportion of purchase will be low because only the consumers with the highest disposable incomes will be able to afford the product.

Contrarily, when the product is free the MU/P ratio is undefined, but under the assumption of rationality, all consumers in the market segment will take only one. The proportion of purchase is one (N/N) when the price equals zero. When the price is some value slightly above zero, the MU/P ratio will be very large but the price will be very low. The product will be affordable to all but one of the consumers in the market segment, and the proportion of purchase will be very high. For this last consumer, demand is perfectly inelastic and the demand function must approach perfect inelasticity asymptotically, just as it must at the upper limit of price. An important implication of this argument is that the proportion of purchase distribution is a function of the distribution of disposable income.

The MU/P ratio can be changed by changing the vari-

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ables that determine the marginal utility of the product. Consider marketing as a demand determinant, ceteris paribus, where marketing includes activities such as sales effort, advertising, promotions, and physical distribution. The purpose of marketing is to inform consumers about the utility of the product and to make it convenient to make a purchase. Marketing influences consumer perceptions of the value of the product (Buzzell & Gale, 1987), and therefore affects the MU/P value.

Assume that the marketing effort can be increased to the point that all consumers are persuaded that the tangible and intangible attributes of the product provide sufficient value to warrant a purchase. The proportion of purchase will equal one. Beyond the point of market saturation, any additional marketing effort will be a waste. As marketing asymptotically approaches this limit, demand with respect to marketing asymptotically becomes perfectly inelastic because demand for the last consumer to buy is perfectly inelastic. With the marketing effort at some value slightly below this maximum, most of the segment will be reached and the proportion of purchase will be very high.

Likewise, there is some minimal marketing effort at which no purchases will be made because no consumer will be persuaded that the MU/P value is adequate to warrant a purchase. The proportion of purchase will equal zero. As marketing asymptotically approaches this lower limit, demand asymptotically becomes perfectly inelastic. At some level of marketing slightly above this minimum the MU/P ratio will be low. Those consumers who are the most aggressive and inquisitive (trendsetters) will buy. Thus for marketing as a determinant of the MU/P ratio, the proportion of purchase distribution will be a function of the distribution of consumer preferences and aggressiveness in satisfying those preferences, assuming that rational consumers will not buy unless the product attributes at least match their preferences. A similar argument can be developed for all other non-price determinants.

The shape of the proportion of purchase distribution as a function of each demand determinant is now known. Each univariate function must asymptotically approach zero when MU/P approaches zero, and must asymptotically approach one when MU/P approaches infinity. To do this the function must have an inflection point. A function, which will meet these conditions, will have a first derivative, which has a maximum, and asymptotically approaches zero at each end of the range of the demand determinant. The second derivative is zero at the point where the first derivative is at its maximum. There are many cumulative probability distributions, which have these mathematical properties. Therefore, the proportion of purchase distribution is a cumulative probability distribution. Note that any cumulative probability distribution with the necessary mathematical properties will satisfy the law of demand.

Probability of Purchase Distribution

The first derivative of the proportion of purchase distribution will be called the probability of purchase distribution. When the probability of purchase function is known, it can be integrated to obtain the proportion of purchase distribution. With the proportion of purchase distribution and the size of

the market segment both known, the demand function can be calculated.

A symmetric probability distribution is a robust assumption for all products except those intended for the segments of the population at either extreme of the population's disposable income distribution. The distribution of consumer preferences in "midrange" market segments is likely to be symmetrically distributed, so that as long as the product is "affordable" the MU/P ratio is likely to be symmetric.

The consumer buys when there is a need, but due to the nature of the product and the life style of the consumer, the interval between purchases is a random variable. It is assumed that the interval between purchases is normally greater than the length of the period for which quantity demanded is being estimated. Therefore, there is some probability, less than one, that any consumer will buy during the period. The proof in this paper does not depend on knowledge of this interval.

The process (proof) is now complete. To derive a demand function theoretically, a probability of purchase distribution as a function of each demand determinant is assumed or obtained empirically. Since consumers will buy the product when the MU/P of the product is greater than or equal to their personal MU/P indifference point, the integral of the probability of purchase distribution is the proportion of purchase distribution. The proportion of purchase at any value of the demand determinant is multiplied by the size of the market segment to obtain the univariate demand function. The product of the univariate demand functions is the market demand function. Quantity demanded is the product of the market demand function and the size of the market segment.

DISCUSSION

The probability of purchase distribution function chosen for this paper is:

$$\text{Prob (F)} = b \frac{e^{bx}}{(e^{bx} + 1)^2} \quad (1)$$

where the variable x has the following forms:

For Price	For Marketing
$\frac{P^S - P}{x = k_1 P_s}$	$\frac{M - M_s}{x = k_2 M_s}$

Note that non-price demand determinants use the same form as for Marketing.

This function was chosen because it is a symmetric distribution, which can be integrated. Therefore, an exact function for the proportion of purchase distribution can be obtained and programmed. The designer does not have to construct a function by choosing elasticities and solving simultaneous equations (Gold and Pray, 1983), by interpolation (Goosen, 1986), or by constructing a function with a particular form (Decker, et al, 1987). Most importantly, a specific

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product, and a specific aspect of consumer behavior can be modeled.

The probability of purchase function (Eq. 1) is a family of curves. The value of the parameter b must be greater than zero. Note that the definition of the variable x takes one form for price and a different form for all other demand determinants. Also note that x is a standardized variable, and that the parameter k is a standard deviation parameter. The parameters P_5 and M_5 are the values for which MU/P for the product equals the average of the consumer's indifference points. Therefore, the definition of the variate x incorporates the indifference concept of ordinal utility.

The proportion of purchase distribution function (the integral of the probability of purchase distribution) is:

$$F_d = \frac{e^{bx}}{e^{bx} + 1}$$

This function is asymptotically bounded between zero and one, and has an inflection point. Therefore the demand function derived from it incorporates the extended law of diminishing returns. The degree of increasing/diminishing returns modeled depends on the value of the parameter b .

The elasticities for a change in the demand determinant equal to one standard deviation are shown in Table 1. These data show that any reasonable elasticity can be modeled at the point of indifference with the proportion of purchase distribution function chosen.

Table 1 Elasticities at $x = 0$

	b			
k	.2	.4	.6	.8
.1	1.00	1.97	2.91	3.80
.2	.50	.99	1.46	1.90
.3	.33	.66	.97	1.27
.4	.25	.49	.73	.95

Table 2 displays the proportional change in quantity demanded as MU/P for the product moves away from the point of indifference by a number of standard deviations. These data clearly show a diminishing returns effect.

Table 2. Proportional Change in quantity demanded.

b	x				
	1	2	3	4	5
.2	.10	.197	.291	.380	.462
.4	.197	.380	.537	.664	.762
.6	.291	.537	.761	.834	.905

While one function was chosen for all demand determinants, this is not a requirement. A different probability of purchase distribution function could be chosen for each demand determinant if desired. The alternatives are a uniform probability distribution function and a skewed distribution function. If a skewed probability distribution function is chosen and it cannot be integrated exactly, the designer can use a numerical approximation algorithm or Goosen's (1986) technique. A uniform probability of purchase distribution can be integrated exactly.

A market demand function of two demand determinants is:

$$Q = Q_s \frac{(F_p)(F_m)}{(F_{ps})(F_{ms})} \quad (3)$$

where F_{ps} and F_{ms} are the proportions of purchase at the indifference point. Any number of demand determinants can be used. The value of Q_s is the quantity demanded when the product's MU/P is equal to the average of the consumers' MU/P indifference points. With a symmetric probability of purchase distribution, the quantity demanded at the average indifference point is equal to one half the size of the market segment.

The market demand function (Eq. 3) has an upper limit on the market volume possible during the simulation. Volume growth can be designed into the simulator by calculating 0~ as growth function. The Product Life Cycle is a likely candidate, but any function which suits the learning objectives of the simulator can be used.

The proportion of purchase distribution function (Eq. 2) can be coded as a subroutine to minimize the computer memory requirement. By designing a simulator with administrator control of the parameters b and k , students can be prevented from "learning the game". Additionally, the developments reported in this paper suggest that designers should publish the values used to establish the indifference point, information related to absolute size of the market segment, the demand growth trend or lack thereof, and the probability of purchase distribution used. If the indifference point is fixed, that should be made known. If it varies, its function, or at least its behavior, should be described.

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