

Developments in Business Simulation & Experiential Exercises, Volume 10, 1983

DEVELOPMENT OF DATA ANALYSIS UNITS DESIGNED TO ENHANCE REASONING AND KNOWLEDGE TRANSFER IN THE COLLEGE LEVEL COURSE

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ABSTRACT

This article presents a method for simulating empirical data for economic decision-making problems. The common cost-volume-profit (CVP) model is the framework. The system can replicate conditions involved in short run or long run optimization situations. The problems developed can be made to appear deterministic or stochastic in nature. A probabilistic approach is illustrated for decision making under uncertainty.

INTRODUCTION

This paper will present a theoretical basis for and a preliminary application of data analysis units designed for knowledge enhancement of economic concepts in the college course. The underlying learning assumption is that knowledge transfer can be enhanced by way of a technique. The unique feature of the approach is the versatility of the potential units which are to be deductively analyzed by the student. The testable hypothesis involves assessment of whether or not the students' transfer of conceptual knowledge is improved. However, at this early stage of development, emphasis will be placed upon the framework for the approach.

The design of the pedagogical economic model to generate the data analysis units incorporates the cost-volume-profit (CVP) approach for the traditional theory of the firm. The CVP approach serves well as a unifying construct for many analyses. Utilizing some elementary statistical concepts and various modifications of the assumptions for the model, data sets conforming to an array of contemporary industrial settings are possible. Elementary courses may utilize the data generating capability of the approach in a deterministic fashion for the certainty case in introductory theory while more advanced courses may utilize the stochastic capabilities for the uncertainty case. By working with the data sets, given specific questions, the student is potentially better able to integrate fundamental economic concepts into a general framework of knowledge.

The next section of the paper discusses the structure of the simulation process through a simple schematic. Section three introduces the theoretical analysis which follows from modifications and extensions of models in the literature. Section four explains the system for generating the data units and indicates the consistency of the procedure with the theoretical base. The fifth section shows the results of student efforts on an early data analysis problem set. The major thrusts of the work are reviewed in the final section.

THE APPROACH

The encyclopedic approach to teaching economics does little to instill the student with critical thinking skills. The instructor is saddled with the task of presenting an expanding body of knowledge to a less sophisticated student

population. The student, all too often, leaves the course with little knowledge that will be transferred to situations meaningful to him. The outcome of the approach at hand, if successful, would be to elevate a vague intuitive feeling on the student's part to a level of formal analysis for each concept so addressed.

The generation of the data sets is very flexible, given the data generating software package utilized. Polynomial expressions up to the fourth power can be specified to create data resembling a wide variety of economic phenomena. Additionally, a stochastic error term can be specified to approximate empirical data. Finally, multiple copies of any data set using the stochastic error term will produce data that is unique yet which still conforms to the general parametric specification of the underlying polynomial. Table one indicates the procedure schematically.

TABLE 1
THE DATA GENERATION PROCESS

$$y = f(x_1^n, x_2^n, \dots, x_m^n; b_0, b_1, \dots, b_m; e)$$

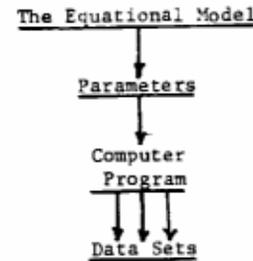
where:

x_1, x_2, \dots, x_m = independent variables

b_0, b_1, \dots, b_m = parameters

e = a stochastic error term

n = any real number



For best results the classroom pedagogy should use good examples, active student participation and real world allusions as well as guidance concerning the axiomatic nature of the economics discipline. The pre-designed data sets enter the process at the point of assigning homework. Each data set is self contained, but may be closely linked to prior or subsequent sets, and has specific questions to be addressed. Through the process of answering the questions by analyzing the data, the student is actively engaging in the process of deductive reasoning. Coupling these student initiated insights with guided instruction and summaries by the instructor, it is argued that the student can become

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better equipped to transfer knowledge to other less familiar situations.

THEORETICAL ANALYSIS

Using a structured simulation approach as a technique requires a theoretical framework as a basis for posing relevant questions and situations. The theory underlying the formulated data units is standard microeconomic principles. The model used as a framework to exhibit the economic principles is the common cost-volume-profit (CVP) analysis.

CVP analysis has been used primarily as a short run optimization technique. Appropriately structured, the CVP model provides information relative to output volume, costs, product prices, and sales volume. The model, even in its simple form, is consistent with the axiom of profit maximization. While the actual use of the CVP approach in business decision making has not been rigorously established, the use of the model as an heuristic device for pedagogical purposes is quite appropriate.

The simple formulation of the model is deterministic i.e., the parameters are assumed to have known values. However, by assuming that certain variables are subject to defined distributions, the model becomes stochastic in nature. The use of stochastic variables allows the application of the CVP model under conditions of uncertainty. The model thus can be utilized to develop risk measures, to illustrate capital asset pricing, and to generally provide a basis for posing long run decision problems. The results of the stochastic model can readily be interpreted in terms of principles of economics.

The Deterministic Version

Elementary CVP analysis specifies the functional forms and assumes that management seeks to maximize short run profits under conditions of certainty. In conformity with much empirical research we specify a linear demand function and a quadratic cost function.

$$\begin{aligned} \text{Let } P &= a - bQ \\ \text{TC} &= d + fQ + gQ^2 \\ \text{TR} &= p \cdot Q = aQ - bQ^2 \\ \pi &= \text{TR} - \text{TC} = -d + (a-f)Q - (b+g)Q^2 \end{aligned}$$

where P = Price
Q = Quantity
TC = Total Cost
TR = Total Revenue
 π = Profit
a, b, d, f, g are parameters

The firm will maximize profits or minimize losses when the

$$\frac{d\pi}{dQ} = \frac{d\text{TR}}{dQ} - \frac{d\text{TC}}{dQ} = 0$$

specifically:

$$\frac{d\pi}{dQ} = (a-f) - 2(b+g)Q = 0$$

$$Q^* = (a-f)/2(b+g) \quad (1)$$

Where Q^* is the profit maximizing level of output for the firm.

The sufficient condition is:

$$\frac{d^2\pi}{dQ^2} = \frac{d^2\text{TR}}{dQ^2} - \frac{d^2\text{TC}}{dQ^2} < 0$$

profit function is at its peak. The necessary condition is:

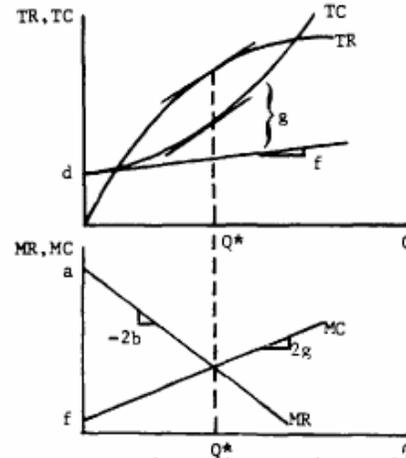
$$\frac{d^2\pi}{dQ^2} = -2(b+g) < 0$$

Alternatively, and in the common parlance of the economist, (1) can be expressed as:

$$(a - 2bQ) = (f + 2gQ) \quad (2)$$

or

Marginal revenue = Marginal cost
It is instructive at this point to plot the functions.



The parameters have interpretive value. For the cost function, d is fixed costs, f is "variable" costs and g is "additional" costs or a sort of marginal cost. Nicely, and due to the quadratic specification of the total cost function, the rate of change of the slope of total cost is a constant $2g$.

The economics of the CVP model implies that to maximize profits, management seeks the output which maximizes the difference between total revenue and total cost ($\text{TR} > \text{TC}$) or where marginal revenue equals marginal cost. The cost function exhibits diminishing returns throughout, which is not unexpected for firms operating within the normal ranges of short run capacity. The revenue function indicates, by its curvilinear nature, that the firm has some price setting power.

The Stochastic Version

Under conditions of uncertainty, even in the short run time period, different behavior is manifest for a risk averse management. The CVP framework can relatively easily accommodate stochastic disturbances in both the revenue and cost functions. For the sake of exposition our approach assumes a stochastic intercept for the demand and total cost functions.

$$\begin{aligned} \text{Let } \tilde{P} &= \tilde{a} - bQ \\ \tilde{\text{TC}} &= \tilde{d} + fQ + gQ^2 \\ \tilde{\text{TR}} &= \tilde{a}Q - bQ^2 \\ \tilde{\pi} &= -\tilde{d} + (\tilde{a} - f)Q - (b + g)Q^2 \end{aligned}$$

where $\tilde{a} = a + \epsilon$
 $\tilde{d} = d + \gamma$

and all other variables are defined as before.

Assume that: $E[\epsilon] = 0$; $E[\gamma] = 0$

$$\sigma_{\epsilon}^2 = \sigma_a^2; \sigma_{\gamma}^2 = \sigma_d^2$$

$$\sigma_{a,d} = \sigma_{\epsilon\gamma} = 0$$

and the distributions of ϵ and γ are multivariate

normal. Expected profit, $\bar{\pi}$, becomes

$$\bar{\pi} = -\bar{d} + (\bar{a} - f)Q - (b + g)Q^2 \quad (3)$$

$$\bar{\pi} = -\bar{d} + \bar{m}Q - nQ^2 \quad (3')$$

where $\bar{m} = (\bar{a} - f)$

$$n = (b + g)$$

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To maximize expected profit, the necessary condition

$$\text{is: } \frac{d\bar{\pi}}{dQ} = \bar{m} - 2nQ = 0$$

$$Q^* = \frac{\bar{m}}{2n} \quad (4)$$

The sufficient condition is:

$$\frac{d^2\bar{\pi}}{dQ^2} = -2n < 0$$

Hence, optimal expected profit becomes

$$\bar{\pi}^* = -\bar{d} + \frac{\bar{m}^2}{4n} \quad (5)$$

after substituting (4) into (3').

Generally, the actual but uncertain profit is:

$$\bar{\pi} = -\bar{d} + (\bar{a} - f)Q - (b + g)Q^2 \quad (6)$$

$$\bar{\pi} = -\bar{d} + \bar{m}Q - nQ^2 \quad (6')$$

where $\bar{m} = (\bar{a} - f)$ and n is defined as before.

Therefore, actual profit, even when management produces Q^* , is still stochastic.

$$\bar{\pi}^* = -\bar{d} + \bar{m}Q^* - nQ^{*2} \quad (7)$$

$$\bar{\pi}^* = -\bar{d} + \bar{m}\left(\frac{\bar{m}}{2n}\right) - n\left(\frac{\bar{m}}{2n}\right)^2 \quad (7')$$

It can be shown that the variance for profit, $\sigma_{\bar{\pi}}^2$, is generally:

$$\sigma_{\bar{\pi}}^2 = \sigma_Y^2 + \sigma_{\epsilon}^2 Q^2 - 2\sigma_{Y\epsilon}Q \quad (8)$$

However, $\sigma_{Y\epsilon}^2 = 0$ by assumption, which makes good economic sense in partial equilibrium micro analysis since price and cost are determined quite apart from one another. Therefore, the expression becomes:

$$\sigma_{\bar{\pi}}^2 = \sigma_Y^2 + \sigma_{\epsilon}^2 Q^2 \quad (8')$$

Then, for the optimal output:

$$\sigma_{\bar{\pi}}^2 = \sigma_Y^2 + \sigma_{\epsilon}^2 \left(\frac{\bar{m}}{2n}\right)^2 \quad (8'')$$

In words, the standard deviation of profit depends on the standard deviations of γ and ϵ as well as on output. As output increases so does the spread of the distribution around profit. However, since the distributions of γ and ϵ are assumed to be multivariate normal, the profit distribution, given output, is also normal.

THE SIMULATION PROCESS

Simulation for business decisions can take many *forms* from simple role playing to complex computer games. Within this milieu, data presentations for students in problem specific situations is an old technique. The data used may be actual empirical data or may be generated by the instructor for a given illustration. Both procedures involve problems of relevancy, consistency, simplicity and production time. Such problems are generally known to persons engaged in the process of education. The simulation approach illustrated below is designed to overcome these problems and retain the theoretical integrity necessary in a simulation approach.

The crux of this

$$P = 6 - .5Q[+ e]$$

$$AVC_1 = Q^2 - 5.48x + 10[+ e]$$

$$AVC_2 = 1.5Q^2 - 5.48Q + 8[+ e]$$

$$AVC_3 = .75Q^2 - 5.48Q + 12[+ e]$$

$$e \sim N(0,1)$$

approach is a computer program that generates data which may appear either deterministic or stochastic in form. The various levels of simulation can be used to illustrate a range of optimization problems. The functional form of the data, the range of the data, and, in cases involving uncertainty, the distribution of the data can be specified.

The data generating software package, as developed, provides the vehicle for the simulations with little loss of theoretical correctness. The package is designed to accept parametric specification inputs (all parameters lie in the realm of real numbers) for up to fourth power polynomials. Randomness of the dependent variable can be simulated by specifying the desired correlation coefficient along with the mean and standard deviation of the error term. The resulting data are then plotted in normalized form and listed in an ordered array. By specifying a correlation coefficient between zero and one, the program approximates the desired fit through an iterative process. Hence, the distribution around the dependent variable is normal while the fit between the dependent and independent variables is specified by the user.

Symbolically: $y = f(x)$; a deterministic model

$y^* = f(x, e)$; a stochastic model

$y = y^* - y = e$

where $e \sim N(0,1)$

Many options present themselves in this interactive program. The user may specify either a normal or rectangular distribution for the independent variable. He may choose the number of observations per each independent variable magnitude. The number of decimal places in the output, order of the data array and whether or not to have a standardized plot of the data are among the other options.

For the CVP framework, the data generating program serves well in deriving stochastic data for the cost and revenue functions. However, as shown in the prior section, the variance of the profit function is directly related to the output level as well as to the variances of the other two functions. The program is not designed to accommodate functional variance specifications for the error term and will tend to understate the variance for the profit function as output rises. We can only recognize this limitation and partially redress it by lowering the correlation coefficient for the profit function data generation below that of the cost and revenue functions. We argue that the loss of theoretical accuracy is minimal in relevant ranges of output and is acceptable for pedagogical modeling.

AN EXAMPLE

The problem listed below was presented to students in an MBA core course at UH/CLC in the Fall, 1982, term. It is the first of many such problems that are to be derived from the processes discussed above. The problem is quite preliminary and reflects only some of the options available. The objective was to provide the students with an economic decision problem involving the optimal size of plant for a firm. The students were to address all questions by analyzing the data, manipulating the data, and interpreting the data from their knowledge of economics. The underlying deterministic formulas are:

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TABLE 2
ECONOMIC PLANNING DATA

<u>OUTPUT</u>	<u>PRICE</u>	<u>AVC1</u>	<u>AVC2</u>	<u>AVC3</u>
.5	5.77	4.00	7.36	9.00
1.0	5.46	3.96	6.64	6.98
1.5	5.13	4.17	5.11	6.25
2.0	5.03	4.08	2.45	4.66
2.5	4.64	3.16	3.10	3.54
3.0	4.56	5.99	2.74	2.53
3.5	4.16	10.19	3.10	2.61
4.0	3.90	11.46	2.74	2.10
4.5	3.61	10.83	6.24	2.91
5.0	3.47	18.61	7.51	3.13
5.5	3.15	25.70	10.87	4.79
5.0	3.11	29.76	12.85	6.25
6.5	2.77	36.03	15.92	8.46
7.0	2.44	39.45	21.04	11.10
7.5	2.17	52.30	24.85	13.31
8.0	2.07	61.14	31.54	16.29
8.5	1.78	69.48	34.54	19.94
9.0	1.45	80.30	39.73	22.55
9.5	1.20	91.59	47.75	28.36
10.0	1.02	103.94	56.09	32.04

WHERE OUTPUT = HOURLY PRODUCTION RATE IN UNITS
 PRICE = MARKET PRICE FOR OUTPUT IN DOLLARS
 AVC1 = THE AVERAGE VARIABLE COST FOR PLANT SIZE 1 IN DOLLARS
 AVC2 = THE AVERAGE VARIABLE COST FOR PLANT SIZE 2 IN DOLLARS
 AVC3 = THE AVERAGE VARIABLE COST FOR PLANT SIZE 3 IN DOLLARS

Illustrative questions:

1. Determine the total variable costs, total costs and marginal costs for plant size two if fixed costs = 1.
2. Determine the marginal product of labor if labor's cost is \$5.00/hour.
3. What is the price elasticity of demand at the optimal output? Is this result consistent with marginal revenue at that point?
4. Provided demand is constant, what is the optimal plant size?

Approximately twenty such questions accompanied the data table. Preliminary results indicate that the majority of students were able to work through the problem with some effort, were forced to review prior economic relationships and were able to integrate knowledge that otherwise would have been untried. These results come *front* a sample of student opinions and are not conclusive.

SUMMARY AND CONCLUSIONS

The simulation procedure incorporates stochastic variables into standard CVP analyses to generate relevant data for problem specific situations. The procedure replicates decision-making under uncertainty as illustrated by microeconomic theory. The data sets can be used to aid students in applying concepts learned in the classroom to situations with real world overtones.

The examples provided were basic, but the procedures are adaptable to complex problems. By assuming multivariate normal distributions, risk considerations and probability measures related to differing levels of profit can be incorporated into the analysis. The student progressively can be introduced to more difficult decision problems within a consistent framework.