Experiential Learning Enters the Eighties, Volume 7, 1980

A METHOD FOR EVALUATING INFORMATION FOR THE EQUIPMENT REPLACEMENT DECISION: AN APPLICATION OF MONTE CARLO SIMULATION

Lyndon E. Coodridge, Alfred University

Problem Statement

ABSTRACT

Nearly every business manager must give careful consideration to the important and recurring problem of equipment investment and/or replacement. Frequently accompanying any Investment/replacement decision are considerations of alternatives such as renting, leasing, and/or custom hiring the necessary equipment capacity. An empirical model was developed to evaluate the relative importance of specific types of information while simultaneously treating alternatives commonly considered in the agricultural equipment investment/ replacement decision process.

The expected value of perfect information (EVPI) about three states of nature was measured In terms of the impact of prior uncertainty on a firms discounted net revenue over a fifteen year planning horizon. The difference between the discounted net revenue received under the condition of prior certainty and that received under the condition of prior uncertainty (given an optimal equipment investment/replacement policy is utilized in both situations) is defined as the EVPI about the state of nature.

INTRODUCTION

Large stockpiles of data have been accumulated for virtually every major form of business enterprise. These make available a source from which information can be extracted for various management decision-making purposes. However, the mere existence of large banks of data, does not guarantee the presence of important information for the decision making function. Only when data can be associated with the solution of a particular problem do they have information content. Thus, a challenge arises for a critical evaluation of any such collections in light of their potential value for specific decision-making functions.

Various segments of information theory contain features which relate directly to decision making in the behavioral sciences. In this study, they were used to develop a framework for determining the value of information in solving an equipment replacement problem in the presence of uncertainty. The approach builds on the original work of Green, et. al.[3] who conducted several experimental games of varying degrees of complexity in the late 60’s. They found that businessmen and academicians often revealed a tendency to seek more information than was absolutely necessary in an attempt to reduce the risk of making a poor decision. A ranking of the relevant information with respect to its value for recurring important decisions can serve as a guide for systematic information acquisition.

To determine the type of information which should be included in a particular system, the recurring problems of major importance to the business operation must be delineated. Following the task of problem delineation, an evaluation of the types of information which will be of most value to the decision maker in dealing with the selected problems can be undertaken.
One of the important recurring problems faced by nearly every type of business operation is that of determining an optional equipment investment and/or replacement policy.

Replacement involves the problem of determining the optimum point in time to replace as well as choosing the best equipment to be purchased for a particular type operation. Replacement decisions also arise due to partial or complete failure of the original unit.

Accompanying any replacement decision are considerations of alternative methods for securing equipment services. Specifically, attention must be given to the possibility of renting, leasing, and/or custom hiring needed equipment capacity.

The purpose of this study is Lo determine the relative importance of specific types of information in dealing with equipment investment replacement decisions.

A Monte Carlo Simulation program written in FORTRAN IV was used to assess the relative importance of specific types of information frequently used as a base for making investment/replacement decisions. The expected value of perfect information with respect to three states of nature was measured in terms of the Impact of prior uncertainty on a firms discounted net revenue over a fifteen year planning horizon.

Prior uncertainty with respect to achieving (1) a specific level of firm growth within (2) a specified time period should have an important bearing on the rent lease purchase and/or custom hire decision for securing equipment services. In determining the amount of equipment investment necessary for a particular task, Information about (3) probable equipment failure rates would prove valuable.

The specific objectives of the study are as follows:

1. Develop a mathematical model for multi-stage equipment investment and/or replacement and simulate a specific set of decision options under alternative sets of environmental conditions.
2. Evaluate information obtained from simulation model changes in order to determine the expected value of specific types of information pertinent to option selection given several specified environmental conditions within which a firm must function.

A brief review of replacement theory as an integral part of the theory of investment provides the necessary framework for building an empirical equipment investment/replacement model.

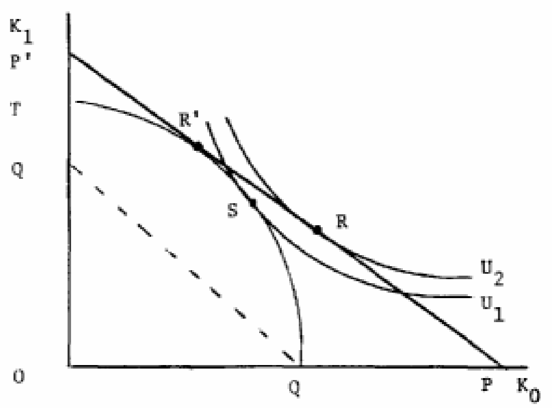
The Equipment Investment Decision in Theory

According to Hirschleifer, “The theory of investment concerns the principle of intertemporal choice--the allocation of resources for consumptive and productive purposes over time”. [4, p. 46j

Implicit in Hirschleifer’s comment is time interdependence with respect to investment decisions. The optimal investment decision can be illustrated in a simplified kinetic (time-variable) situation involving two transformation periods under conditions of a perfect capital market.

A perfect capital market refers to a situation where (1) borrowing and lending rates of interest are equal, (2) the entrepreneur may borrow and lend without restriction, and (3) the rate of interest is not affected by the amounts borrowed or lent.

FIGURE 1. DETERMINING THE OPTIMAL INVESTMENT DECISION WITH TWO TRANSFORMATION PERIODS.



In Figure 1. , (%) represents the amount of actual or potential income in the first period. The actual or potential income in the second period is represented by

K_1). The curve “QST” shows the production investment opportunity contour of the firm. The production investment opportunity contour represents the locus of all points attainable to the entrepreneur as he sacrifices more and more of KO by productive investments yielding Kl in return, given his capital position. QST represents a sequence of projects arranged so as to start with the one yielding the highest rate of return at the lower right and ending with the lowest rate of return encountered when the last dollar of period one is sacrificed at the upper left. The problem facing the entrepreneur is to choose an optimal time pattern for investments among the opportunities available.

Assume the entrepreneur has a utility function which relates the income in each of the two periods. This preference function is represented in Figure 1 by the curves U_1 and U2.

Since the entrepreneur would decide to move to the highest indifference curve, he would move along his production investment opportunity curve to point “S". En the absence of a capital market, "S" would be the optimal investment. However, in the presence of a perfect capital market as assumed, the entrepreneur may move further on QST to R and from this point along the market line PP’ to point R by borrowing additional funds. The slope of the market line PP’ is given by 1 where i represents the borrowing rate of interest. l+i

Point R represents the optimal investment decision since it is on the highest indifference curve.

Hirschleifer compared the above solution with the commonly used present value rule. The present value formula for the two period analysis may be represented as follows:

$$PV = K_0 + K_1 (1+i)^{-1} \quad (1)$$

where PV = present value of income derived from machinen investments.

i = rate of interest.

If the entrepreneur invests in the first period sacrificing present (KO) for future income (K_1) he will receive (in the two period case) in the second period

\+ K_1 (l+i)-l.

The entrepreneur attaining the solution R in Figure 1 will have maximized present value of income in terms of the productive investment decision, With the productive set QST available, the choice R’ is on the highest market line attainable (PP’). Since market lines are lines connecting combinations of equal present value (or alternatively income is the present value measure of the combinations along the market line) the highest attainable income is that corresponding to the line PP’. This indicates that “the rule directing maximization of overall present value is correct in this case if the entrepreneur in attaining R’ will actually forego present (KO) for future income (K_1) so as to reach point R.” [4, pp. 62-63j

<u>The Equipment Replacement Decision in Theory</u>

Replacement theory is an integral part of the theory of investment. It is concerned with investment in capital goods whose life is a decision variable. Modern replacement theory gained considerable momentum in 1923 when J. S. Taylor [8, pp. 29-30] developed, by means of a discrete period analysis, a formula relating the average unit cost of the output of a machine over L years to each of the following factors:

1. The acquisition cost of the new machine.
2. The scrap of salvage value of the machine after L periods of service.
3. The machine’s operating costs in each period of service up to the 4th Period.
4. The output of the machine in each period.
5. The rate of interest.

Taylor then determined the appropriate length of time “L” to keep a machine so as to minimize the unit cost of production.

Hotelling, in 1925, promoted the argument that it is not the objective of the entrepreneur to minimize the unit cost of production. The entrepreneur’s main desire is to maximize the present value of the machine’s output minus its operating costs plus its salvage value at the age of the machine at time “L” of replacement. [5, p. 341] The “fundamental formula” that Hotelling advanced to meet his objective may be written in the following form:

V(T) = T CR(t) E(t)} e-r(t-T) dt + S(L) e-r(L-T) (2)

V(T) is the value of equipment at time T.

S(L) is a function giving the scrap value at time L.

R(T) is the machine’s revenue at time t.

E(t) is the machine’s operating costs at time t, and r is the continuous rate of interest. [8, p. 1291

The solution requires finding L such that V(T) is at a maximum.

In 1940, Preinreich advanced the argument that the economic life of one machine could not be determined in and of itself but rather consideration must be given to subsequent machines throughout the firm's planning horizon. [8, p. 1291 It was proposed that the present value of the earnings of the future machine replacements minus the present value of the costs of all such machines was to be maximized. That is, the following

$$F + \sum_{k=0}^{r} e^{-rkL} \{ \int_0^L \{R(t)-E(t)\} e^{-rt} dt + S(L)e^{-rL} - P\} \quad (3)$$

P = Lhe acquisition cost of each new machine in the replacement chain “k" and the remaining variables are the same as those identified in Hotellings’s model.

The inclusion of the acquisition cost of each new machine in the replacement chain was deemed necessary due to the fact that the existing machine must compete with all subsequent replacements. His proposition, founded on the opportunity cost concept, was an important one which was absent in explicit form from previous replacement models.

Finally, Terborgh in 1949 [8,p. 1291 extrapolated the historical rate of obsolescence into the future on the principle of a uniform rate of technological discovery. This was a unique contribution in that prior analytical pursuits failed to explicitly include obsolescence. For example, Hotelling merely considered obsolescence as an increment to operating cost which should be insurable much the same as fire, theft, etc.

In Terborgh’s model, the costs of operating inferiority (an increasing cost with age), and capital cost, (a decreasing cost with age) were sutmued (E) per year as follows:

$$E = \frac{(n-1)c}{2} + \{\frac{A-S}{n} + \frac{r}{2}(A+S)\} \quad (4)$$

c = the constant rate of cost accumulation due to operating Inferiority (the inferiority gradient)
A = the acquisition cost.
S = the salvage value.
r = the rate of return of capital investments within the firm.
n = the age of the equipment.

for the most part, been taken for granted. That is, the appropriate replacement criteL ion should be the

The average operating cost increase per year due to inferiority $\frac{(n-1)c}{2}$ in equation (4) was obtained as follows:

$$\frac{0+C+2C+\ldots+(n-1)C}{n} = \frac{1}{n}\left(\frac{n(n-1)C}{2}\right) = \frac{(n-1)C}{2} \quad (5)$$

The average investment per year in (4) is $\frac{A-S}{n}$.

maximization of the present value of the machine’s net revenue.

The replacement decision must be made in view of the alternative options available for securing needed equipment capacity. That is, the total equipment

$$\frac{dE}{dn} = \frac{C}{2} - (A-S)n^{-2} = 0 \quad (6)$$

Hence N_0, age that minimizes the sum of operating inferiority and capital costs is

$$N_0 = \left[\frac{2(A-S)}{C}\right]^{\frac{1}{2}} \quad (7)$$

investment each year is a function of the expenditures for rented, leased, and/or custom hired services as well as the expenditures for replacement machines. As indicated by Hirschleifer’s framework, the combination and sequence of alternative investments should be arranged so as to enable the entrepreneur to reach his highest market line (Figure 1). [4, p. 62] If the entrepreneur does have the investments so arranged, and if he is willing to forego present for future income, the optimal time pattern for

where:

investments (among the opportunities available) can be achieved when the present value rule is in effect.

<u>The Relevant Planning Horizon for Simulation</u>

In choosing the optimal time pattern for investment and/or replacement, the appropriate length of planning horizon becomes a central issue. For example, Hirschleifer has indicated that the optimal first period investment decision can be achieved only if the possibility of sacrificing present for future income is recognized. [4, p. 63j In terms of replacement, Preinreich has indicated that subsequent machines throughout the firm’s planning horizon must be considered in the initial purchase decision. [7j Consequently, to make the correct first period decision, the relevant planning horizon for simulation purposes must be of sufficient length so the arbitrary evaluation at the end has no effect on first period decisions.

There are two features of the planning environment that provide intuitive guidelines for determining the appropriate length of horizon. Both the lesser utility of future earnings as indicated by the present value concept and the growing uncertainty about the planning coefficients in the more distant periods tend to be horizon shorteners.

Based on the features of the planning environment, a fifteen year planning horizon was judged to be long enough for simulation purposes so that arbitrary evaluation at the end would, in all probability, have no effect on first-period decisions. Of course, the validity of the judgment can be checked with sensitivity analysis.

<u>The Value of Information in the Decision Making Process</u>

When the term "value" is associated with a particular type of information, an implication arises that the information improves the outcome of the decision making process.

In this study, the interest lies in whether a particular type of information is helpful in the complex process of choosing among alternative methods for obtaining equipment services. Its value is measured in terms of expected monetary payoff associated with a particular investment/replacement strategy in short, a TiBayesian approach”.

Five states of nature are considered in this study. Each state of nature is represented as Oij where i(i = 1. . . .5) refers to the state of nature being considered, and j(j = 1 or 2) is the unique value associated with the state of nature.

The last term r/2 (A+S) is an approximation of the average yearly cost of (A-S) dollars for a period of n-1 years on which payments are made annually p1us the cost of borrowing S dollars in n years.

The optimal age to minimize “E” is calculated as follows:
In essence then, the average increase in costs due to operating inferiority is equal to the square rooL of the average annual depreciation in capital at the optimum. In later work Terborgh considers service values, taxes, salvage values, and alternative forms of investment as well as obsolescence. [2, p. 334]

The four contributions mentioned above brought the theory of replacement to a fairly complete form. In fact, since the two initial papers by Taylor and Hotelling, the

If $P(\Theta_{i1}) = a$, then $P(\Theta_{i2}) = (1-a)$ for $i = 1....5$.

criterion for replacement decisions has, Specifically, to obtain the optimal Bayesian strategy for securing equipment services, the following procedure is used:

1. The states of nature are defined to describe an agricultural setting as follows:

O11 = a 75 percent increase (over the planning horizon) in the initial firm size. The initial size of the firm is defined in terms of the nuther of acres of land involved in the production process.

O12 = a 25 percent increase (over the planning horizon) in the initial firm size.

021 = a firm expansion in year three of the 15 year planning horizon.

®22 = a firm expansion in year nine of the 15 year planning horizon.

O31 = a normal level of machinery failure. That is, a normal amount of time loss from field operations and a normal level of costs associated with machinery repair. "Normal" time losses ate defined in terms of maximum likelihood estimates derived from published engineering statistics.

®32 = a 25 percent above normal level of time and monetary costs associated with equipment failure.

041 = a high prevailing interest rate of 14 percent.

042 = a low prevailing interest rate of 9 percent.

O51 = a high level of risk preference. The desire of the entrepreneur to complete all field operations every seven out of ten years in the 15 years firm planning horizon.

052 = the lower level of risk preference, the desire to complete all operations nine out of every ten years in the planning horizon.

2. In successive experiments, prior probabilities of 1.0, 0.75, 0.25, and 0.0 are placed on Oil (for i = .5) . An experimental design is constructed so as to isolate the main effects of changing a state of nature value while holding all other states of nature constant.

3. For each experiment (given the probability assigned to each state of nature) the expected monetary payoff as a result of following a particular machinery investment/replacement strategy is calculated. That is, the discounted net revenue associated with each strategy is obtained. Each machinery investment and/ or replacement strategy is defined in tens of one level of the following eight courses of action:

Al = custom preparing and planting of (0 100) percent of the tillable acreage each year.

A2 = custom harvesting of (0 100) percent of the cropland each year.

A3 = securing machinery service by purchasing (0 4) tractors of sizes (1 5).

A4 = purchasing (0 4) combines of sizes (1 5).

AS = leasing (0 4) tractors of sizes (1....5).

A6 = leasing (0 4) combines of sizes (l....5).

A7 renting (0 4) tractors of sizes (1... .5).

A8 = renting (0 4) combines of sizes (1....5). All sets (Al. . . A8) are considered in combination, representing a complete strategy. Each of the methods for securing machinery services as well as the number and sizes of machines available were selected. That is, a strategy refers to a particular combination of levels set for the eight courses of action.

4. The strategy leading to the highest expected monetary payoff (the highest 15 year expected discounted net revenue)

$$G_1 = \sum_{\ell=1}^{8}\Big[G(A_\ell/\Theta_{i1})\cdot P(\Theta_{i1}) + G(A_\ell/\Theta_{i2}.P(\Theta_{i2})\Big] \qquad (8)$$

where $P(\Theta_{i1}) = \Big[1 - P(\Theta_{i2})\Big]$ and $i=1.....5$.

is calculated in the computer simulation program.

The expected discounted net revenue gained from a particular strategy may be calculated as follows:

S. The value attributed to information relevant to a particular state of nature is defined in terms of the cost of prior uncertainty. The cost of prior uncertainty is expressed as the loss in expected discounted net revenue over the fifteen year planning horizon from the assignment of a prior probability of less than unity to a state of nature value.

$$\text{EVPI} = \sum_{\ell=1}^{8}\{\ G(A_\ell/\Theta_{i1}) \ - \Big[G(A_\ell/\Theta_{i1})\cdot P(\Theta_{i1}) + G(A_\ell/\Theta_{i2}).P(\Theta_{i2})\Big] \qquad (9)$$

where $P(\Theta_{i1}) < 1.0$, $P(\Theta_{i2}) = 1 - P(\Theta_{i1})$, and $i = 1....5$.

The expected value of perfect information (EVPI) relative to each state of nature may be calculated as follows:

The above theoretical framework is used to obtain the expected value of perfect information relative to the various states of nature considered in this study.

The difference between the discounted net revenue received under the condition of prior certainty and that received under the condition of prior uncertainty (given an optimal machinery investment/replacement mix is utilized in both situations) is defined as the expected value of perfect information about that state of nature.

<u>The Empirical Model</u>

The objective function employed in this study is designed to maximize the present value (as determined by the borrowing rate of interest) of the net revenue stream of the firm over a fifteen year planning horizon.

$$\text{Max } G = \sum_{t=1}^{T}\Big[(P_{yt} \cdot Y_t/(1+d)^t) \ - \sum_{m=1}^{M}(P_{mt} \cdot X_{mt}/(1+d)^{t-1})$$

Discounted Fixed Costs Discounted Purchase Price

$$- \sum_{c=1}^{C}(Q_{ct}/(1+d)^{t-1})\Big] \ - \sum_{n=1}^{N}\ \sum_{j=1}^{J}\Big[P_{nj}K_{nj}/(1+d)^{RT_{nj}}$$

$$- S_{nj}K_{nj}/(1+d)^{RT_{nj}}\Big] \qquad (10)$$

where G = total discounted net revenue for the fifteen year firm planning horizon.

t = year 1.....T (T=15).

P_{yt} = Price (P) of output (y) in year (t).

Y_t = Total output of (y) in year (t).

d = Discount rate based on the borrowing rate of interest ($d = d(\Theta_{41}$ or $\Theta_{42})$).

P_{mt} = Price (P) of variable resource (m) used in year (t).

X_{mt} = Units (X) of variable resource (m) used in year (t).

Q_{ct} = Fixed cost (Q) of capital resource (c) in year (t).

P_{nj} = Price (P) of jth durable resource (machine (n)).

The objective function may be represented as follows: Discounted Gross Revenue Discounted Variable Costs Discounted Salvage Value

K_{nj} = Number (K) of jth machines (n).

S_{nj} = Salvage value (S) of jth machines (n).

RT_{nj} = The year (RT) the jth machine (n) replaced.

Each of the preceding terms is a function of the particular set of acts, A_ℓ, where ℓ= 1.....8, and the prevailing states of nature, Θ_{ij}, where i = 1.....5, and j = 1 or 2. The functional relationships utilized in the simulation model may be represented as follows:

Discounted Gross Revenue		Yields Per Acre	Acres Harvested	Product Price	
$P_{yt}Y_t/(1+d)^t$	=	$f_1(Y(t),$	$AH(t),$	$P_y(t))$	(11)

Yields Per Acre		Week Planted	Day Harvested	Yield Trend	
$Y(t)$	=	$f_2(WP(t),$	$DH(t),$	$YT(T))$	(12)

Week Planted		Day Harvested	Total Interruption	
$WP(t)$	=	$f_3(DH(t-1),$	$TI(t))$	(13)

Day Harvested		Week Planted	Total Interruptions	
$DH(t)$	=	$f_4(WP(t),$	$TI(t))$	(14)

Total Interruptions		Weather Interruptions	Machinery Failure	
$TI(t)$	=	$f_5(WI(t),$	$MF(t))$	(15)

Weather Interruptions

$$WI(t) = f_6(\Theta_{ij}) \tag{16}$$

where (i=5) and (j=1 or 2).

Machinery Failure

$$MF(t) = f_7(A_\ell, \Theta_{ij}) \quad \text{where } (\ell = 3...8),\ (i = 3) \text{ and } (j = 1 \text{ or } 2) \tag{17}$$

Discounted Variable Costs		Repair Costs	Fertilizer Costs	Lime Costs	Seed Cost
$P_{mt}X_{mt}/(1+d)^{(t-1)}$	=	$f_8(RC(t),$	$FC(t),$	$LC(t),$	$SC(t)$

Labor Cost	Fuel Cost	Courses of Action	Other Costs		
$LA(t),$	$FU(t),$	$A\ell,$	$OC(t))$	where ℓ = (1,2,(5...8))	(18)

Repair Costs		Courses of action	States of Nature	
$RC(t)$	=	$f_9(A_\ell,$	$\Theta_{ij})$	(19)

where ℓ= (3,4), i = (3) and j = (1 or 2)

Labor Costs

$$LA(t) = f_{10}(A_\ell) \quad \text{where } \ell = (3....8) \tag{20}$$

Fuel Costs

$$FU(t) = f_{11}(A_\ell) \tag{21}$$

Discounted Fixed Costs		Taxes	Interest	Insurance Cost	Housing Cost	
$Q_{ct}/(1+d)^{(t-1)}$	=	$f_{12}(TX(t),$	$IN(t),$	$IC(t),$	$HC(t))$	(22)

Taxes		Total Acreage	
$TX(t)$	=	$f_{13}(AA(t))$	(23)

Acreage State of Nature

$$AA(t) = f_{14}\ (\Theta_{ij}) \qquad \text{where } i = 1..2 \text{ and } j = 1 \text{ or } 2 \tag{24}$$

Interest States of Nature

$$IN(t) = f_{15}\ (\Theta_{ij}) \qquad \text{where } i = 4 \text{ and } j = 1 \text{ or } 2 \tag{25}$$

Insurance Cost Courses of Action

$$IC(t) = f_{16}\ (A_{\ell}) \tag{26}$$

Housing Courses of Action

$$HC(t) = f_{17}\ (A_{\ell}) \qquad \text{where } \ell = 3,4 \tag{27}$$

Discounted Purchase Price Machine Size Year Purchased

$$P_{nj}K_{nj}/(1+d)\ \ RT_{nj} = f_{18}\ \ (MS(t), \quad YP(t)) \tag{28}$$

Discounted Salvage Value Machine Size Year Purchased Year Sold

$$S_{nj} - K_{nj}/(1+d)\ \ RT_{nj} = f_{19}\ \ (MS(t), \quad YP(t), \quad YS(t)) \tag{29}$$

Machinery Size Courses of Action

$$MS(t) = f_{20}\ (A_{\ell}) \qquad \text{where } \ell = (3,4) \tag{30}$$

Year Purchased Courses of Action

$$YP(t) = f_{21}\ (A_{\ell}) \qquad \text{where } \ell = (3,4) \tag{31}$$

Year Sold Courses of Action

$$YS(t) = f_{22}\ (A_{\ell}) \qquad \text{where } \ell = (3,4) \tag{32}$$

Equation (11) is maximized subject to the following acreage and time constraints:

Acres Harvested, Acres Planted, Acres Disked, Acres Plowed, Total Acreage

$$AH(t) \le APL(t) \le AD(t) \le AP(t) \le AS(t) \tag{33}$$

Within any week (w) where $1 \le w \le 4$:

Time to plow, Time to disc, Time Available

$$TP(w) + TD(w) \le TIM\ (w), \text{ and} \tag{34}$$

Acres Disked, Acres Plowed

$$\sum_{w=1}^{4} AD(w) \le \sum_{w=1}^{4} AP(w) \tag{35}$$

Within any week (w) where $5 \le w \le 10$:

Time to Plow, Time to Disc, Time to Plant, Time Available

$$TP(w) + TD(w) + TPL(w) \le TIM(w), \text{ and} \tag{36}$$

Acres Planted, Acres Disked, Acres Plowed

$$\sum_{w=5}^{10} APL(w) \le \sum_{w=5}^{10} AD(w) \le \sum_{w=5}^{10} AP(w) \tag{37}$$

Within any week (w) where $25 \le w \le 34$:

Time to Harvest, Time to Disc, Time to Plow, Time Available

$$THA(w) + TD(w) + TP(w) \le TIM(w), \text{ and} \tag{38}$$

Acres Plowed, Acres Disked, Acres Harvested

$$\sum_{w=25}^{34} AP(w) \le \sum_{w=25}^{34} AD(w) \le \sum_{w=25}^{34} AH(w) \tag{39}$$

Solution Technique

The Monte-Carlo prograunning technique employed to solve the machinery investment/replacement model begins with the random selection of various courses of action. A particular set of acts (courses of action) define a specific machinery investment/replacement strategy.

If the course of action which includes machinery investment and/or replacement is randomly selected among the alternatives available for securing needed machimery capacity, the number of machines, size of each machine, and years the machines are to be placed in solu- tion is randomly selected. Rented and/or leased machines are randomly selected.

Following the random selection of the various sizes, ages, and types of machines that are to be included over the fifteen-year planning horizon, the business operation is simulated using the machines which were selected. The discounted net revenue that is associated with a particular fifteen-year machinery investment/replacement strategy is then revealed.

Reduction of Decision Space

Because of the large number of courses of action (possible machinery investment/replacement combinations) being considered in this study, an evaluation of every possible combination in order to obtain the optimal strategy is neither necessary nor practical in terms of processing cost. The number of 15-year equipment investment/replacement strategies (Az)s that need to be evaluated for each specified environment (set of

is directly related to the precision required. That is, if the desired probability of obtaining a maximum response is set at .95 (S= .95) and “a” is defined as .05, then the corresponding number of machinery combinations required is 59 (n59).

TABLE 1. NUMBER OF OBSERVATIONS REQUIRED WITH RANDOM MAXIMUM-SEEKING SOLUTION METHOD.

“a”		"S"		
.20	.80	.90	.95	.99
	8	11	14	21
.10	16	22	29	44
.05	32	45	59	90
.025	64	91	119	182

In terms of this study then, if “S” is set at .95, then it would require an evaluation of 59 machinery combinations to be 95 percent confident of finding that combination which reveals a discounted net revenue which is within five percent of the true optimum. The impact of precision “a” and confidence level “S” on the number of trials simulated to achieve a specific level of accuracy may be ascertained from table 1.

The solution procedure as described will reveal the best solution with a specific value assigned to the states of nature being considered. However, as previously indicated, five states of nature are considered in this study: (1) level of growth, (2) year of land acquisition, (3) level of machinery failure, (4) interest rate level, and (5) risk preference. Each state of nature will take on only one of two possible values. Furthermore, alternative prior probabilities are assigned to the level of growth, year of land acquisition, and level of machinery failure.

Therefore, to isolate the impact of changing the state of nature values as well as ascertain the change in discounted net revenue brought about by alternative prior probabilities assigned to Items (1), (2), and (3) mentioned directly above, a total of 256 combinations would need to be evaluated. That is, a total of 59 different machinery combinations for 256 combinations of the state of nature values would be required.

In final analysis 15,104 complete fifteen-year strategies would be generated. It is not practical, nor is it necessary at this point either to calculate the effects of this many combinations.

A 1/16 . 28 fractional factorial experimental design may be employed to select a specified percentage of Lhe total combinations from which the entire response surface may be generated.

The 1/16 . 28 fractional factorial experimental design requires sixteen observations to estimate the impact of each of the states of nature (and their associated prior probabilities) on the firm’s discounted net revenue.

The experimental plan is presented in Table 2. The key to Table 2 is given in Table 3.

A total of 59 strategies (different methods for securing the total needed machinery capacity over the fifteen-year planning horizon) is obtained for each point (case) on the experimental design. That particular machinery investment replacement strategy which reveals the highest expected discounted net revenue (out of the 59 strategies evaluated per case) is considered to be the optimal (best) strategy given the prevailing states of nature.

The Response Surface

Finally, by employing an ordinary least squares regression technique, with the Discounted Net Revenue (obtained from the optimal solution to each of the 16 cases) as the dependent variable and the various states of nature as the independent variables (with the probability of each state of nature occurring set at unity) the impact of each of the states of nature on the firms total discounted net revenue is obtained. By lowering the prior probability levels to less than unity (.75 as opposed to 1.0) for three particular states of nature (growth rate, growth timing, and level of machinery failure) the impact of prior uncertainty of the firms discounted net revenue is revealed.

Specifically, in order to isolate the main effects of each of the above states of nature (and their associated prior probabilities) , the discounted net revenue is represented as a function of each of the five states of nature (three of which have probabilities of less than unity attached). This is expressed by means of an orthogonal polynomial equation. The impact of each state of nature and prior uncertainty on the firms discounted net revenue may be obtained as follows:

$$G = a + b_1\Theta_{1j} + b_2\Theta_{2j} + b_3\Theta_{3j} + b_4\,\Theta_{4j} + b_5\,\Theta_{5j} + b6(P(\Theta_{1j})) + b_7(P(\Theta_{2j})) + b_8\,(P(\Theta_{3j})) \qquad (40)$$

where $j = 1$ or 2

Where G = observed discounted net revenue,
a = grand mean,
$b_{1....8}$ = estimated parameters,
$\Theta 1j.....5j$ and $P(\Theta_{1j}).....P(\Theta_{3j})$ are as previously defined in table 3.

The impact is defined in terms of the expected gain in discounted net revenue due to a reduction prior uncertainty concerning the states of nature in question which is the expected value of perfect information.

TABLE 2. EXPERIMENTAL DESIGN FOR ALTERNATIVE STATES OF NATURE

	Firm	Year of	Machinery	Rate	Level	Prior Probabilities		
						Firm	Year of	Machinery
	Growth	Land	Failure	of	of	Growth	Land	Failure
Case	Level	Purchase	Level	Interest	Risk	Level	Purchase	Level
1	1	1	1	1	1	1	1	1
2	0	0	0	0	0	0	0	0
3	0	0	1	1	1	0	0	1
4	1	1	0	0	0	1	1	0
5	1	0	1	0	0	1	0	1
6	1	0	0	1	1	1	0	0
7	0	1	1	0	0	0	1	1
8	0	1	0	1	1	0	1	0
9	1	0	1	1	0	0	1	0
10	1	0	0	0	1	0	1	1
11	0	1	1	1	0	1	0	0
12	0	1	0	0	1	1	0	1
13	1	1	1	0	1	0	0	0
14	1	1	0	1	0	0	0	1
15	0	0	1	0	1	1	1	0
16	0	0	0	1	1	1	1	1

The one and zero in the above experimental plan refer to levels placed on the various states of nature being considered.

TABLE 3. KEY TO VALUES IN THE EXPERIMENTAL DESIGN.

States of Nature	1	0
Level of Firm Growth	high (Θ_{11})	low (Θ_{12})
Year of Land Acquisition	three (Θ_{21})	nine (Θ_{22})
Machinery Failure Level	low (Θ_{31})	high (Θ_{32})
Rate of Interest	low (Θ_{41})	high (Θ_{42})
Level of Risk Preference	high (Θ_{51})	low (Θ_{52})
Prior Probabilities		
Firm Growth Level	$P(\Theta_{11})=.75$	$P(\Theta_{12})=.25$
Year of Land Purchase	$P(\Theta_{21})=.75$	$P(\Theta_{22})=.25$
Machinery Failure Level	$P(\Theta_{31})=.75$	$P(\Theta_{32})=.25$

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