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AN EXPERIENTIAL EXERCISE IN

MULTIDIMENSIONAL SCALING

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Multidimensional Scaling (MDS) is a set of multivariate analysis techniques widely used in behavioral and marketing research. It has been the subject of numerous books, papers, and journal articles. However, it is also a technology that appears rather mysterious to both managers and students, who will become tomorrow's managers.

The purpose of the exercise described in this paper is to provide students with a comprehensible, hands-on, MDS experience. The experience is in two parts. The first is the recovery of a geometric figure designed by the student. The second is an actual application of MDS by the student with himself as the respondent.

MDS APPLICATIONS

There are two major uses of MDS technology--perceptual mapping and preference mapping. Perceptual mapping is based upon a respondent's judgments about the similarities or differences between pairs of objects. Input consists of a matrix of these paired judgments of perceived similarities or differences. The output of the scaling routine includes a graphic display, or "perceptual map." The map is constructed so that objects which are perceived as more similar are plotted closer together than other objects which are perceived as more dissimilar.

Preference mapping is based on rank orderings of objects. A common application is to have a number of respondents rank order the same set of objects. The scaling routine produces a graphic representation of the objects similar to perceptual mapping. The "map" also plots each respondent's "ideal" point on the graph. Another application of preference mapping is to ask a single respondent to rank order objects on several different attributes. The resulting map displays both the objects and

"ideal" values for each attribute.

TEACHING MDS

The literature contains many fine examples of both perceptual and preference scaling which may be used as a basis for classroom instruction. However, the relationship between paired judgments or preference rankings and the graphic representations is not intuitively obvious. Two credibility problems typically exist in the minds of students. First, do the maps really display the information contained in the input data? Second, are the maps really consistent with the perceptions or preferences of the respondents?

The exercises described in this paper are designed to provide students with comprehensible answers to these two questions through actual experience with MDS. The first exercise asks the student to prepare paired comparison data for a known geometric figure. Since the paired distance measures are prepared from the figure by the student, the relationship between the geometric shape and the input data for the scaling is known. The ability of the MDS routine to recover the geometric configuration provides the student with an intuitive understanding of what the scaling routine does (the reverse of the measurement process done by the student). The exercise demonstrates that the geometric representation produced by the scaling does contain the information embedded in the paired comparison matrix.

The second exercise presents the student with an opportunity to test the congruence between MDS output and a respondent's perceptions and preferences by questioning a subject whose beliefs and attitudes are known, the student himself. The exercise asks the student to answer paired similarity questions about product brands with which he is familiar. MDS programming then produces a perceptual map based on the student's responses. The exercise continues with several questions asking for rank orderings of the brands on different product attributes. MDS is

used again to produce a preference mapping relating the brands and the product attributes.

THE MDS PROGRAM

A fairly large number of computer programs are available for both perceptual and preference scaling. The most widely known are available from Bell Labs in Princeton, New Jersey. One such program is KYST which is a merge of the perceptual scaling program TORSCA and the preference scaling program MDSCAL. KYST is convenient for this exercise not only for its ability to do both perceptual and preference scaling, but also because program control statements are coded in English and, therefore, easily learned by students.

At many computer centers it is possible to circumvent the need for requiring the students to learn the mechanics of operating the scaling programs by storing the necessary control statements on the computer library. The examples of the exercises described below go one step further. The exercises are presented in the form of a conversational computer program which provides instructions, appropriate questions, and scaling results. An advantage of such a program, at installations where it is feasible, is that the student is provided with nearly instant feedback to his responses which reinforces the learning experience.

FIGURE RECOVERY

Figure 1 is a geometric design based, roughly, on the letter ‘A’. The design contains eight reference points designated by the letters ‘A’ through ‘H’. Students are asked to prepare a geometric design of their own choice with eight reference points similar to Figure 1.

The next step is to measure the distances between the eight points. Values for Figure 1 are given in Table 1. The values were computed from a graph paper plot of Figure 1 upon which horizontal and vertical distance between points could be measured

rather accurately. Straight-line distances were computed using the Pythagorean Theorem.

Figures 2 and 3 are a specimen of the MDS program based on the design in Figure 1. The "@ADD" instruction at the top of Figure 2 is all that is necessary to execute the program and assign the required data files containing the questionnaires. The same program is used for the self-administered perceptual and preference mapping as well as the Figure recovery.

The data from Table 1 are entered as shown at the bottom of Figure 2. Figure 3 contains the MDS result. The connecting lines were drawn in by hand to highlight the relationship between Figures 1 and 3. Rotation of the original design by the MDS program is not unusual; the similarity between the original design and the recovery is self-evident.

PERCEPTUAL AND PREFERENCE MAPPING

Figures 4 through 8 contain an example of self-administered perceptual and preference mapping. Since a successful result is considerably more likely if the respondent has definite opinions and feelings about the objects considered, students have a choice among four product categories for this phase of the exercise--brands of beer, automobile models, laundry detergents, and brands of soda (soft drinks). The example is for laundry detergents.

After all of the paired comparison questions have been answered, the respondent has an opportunity to correct any judgments he is dissatisfied with, and the scaling program is executed. The scaling program represents the objects alphabetically so the questionnaire program provides a symbol directory. Since two objects may plot at virtually the same point on the graph, the plot routine provides coordinate values for these proximate points so that the underlying objects may be identified.

The scaling program provides a calculation of Kruscal's stress measure which indicates the degree of inconsistency between the data and the final configuration.

The message that a "satisfactory stress was achieved" indicates that the solution is probably a meaningful representation of the respondent's judgments.

The second part of the exercise asks the student to rank order the brands on five attributes believed relevant to brand preference and perceived differences between brands. Students are urged to study the perceptual map from the first part of this exercise as a guide to these rankings, but it is not essential that they do so. If the assumption that there is an interrelationship between preferences for attributes, perception of brands, and preferences for brands is true, then the rank orderings should be much the same regardless of whether or not the respondent studies the first plot.

Again, the student may correct responses he is dissatisfied with before the scaling program is executed. MDSCAL does not guarantee a global minimum will be achieved on Kruscal's stress measure. Therefore, it is important that the MDS program be provided with a rational starting configuration. The program uses the configuration obtained in the perceptual mapping phase of the exercise. Two plots are produced. In the first, the relative positions of the brands are held fixed from the perceptual mapping phase and the attributes are fit into the configuration. In the second plot, the rank order data are taken as additional information and the program is allowed to modify the perceptual relationships as part of the fitting process.

At the conclusion of the exercise, the program automatically prints additional detailed output from the scaling program on the computer center's line printer. This output contains a summary of the calculations used to produce the scaling solution and coordinate values for all of the points plotted.

As part of the assignment, students are asked to evaluate the configurations produced. Of major interest is: are the results credible in terms of the student's own thinking about the comparison judgments and rank orderings at the time he made them? Since results are made available almost immediately by this version of the

program, the student is unlikely to have forgotten what he was thinking about in considering his responses. Therefore, it should be possible to interpret the reasons for the positions of the points on the plots. It may also be possible to identify differences in the relative positions of the brands that are based upon attributes not asked about in the rank order questions.

CONCLUSIONS

The experiential learning application described in this paper is a direct product of faculty research projects. At the time this manuscript was being pre- pared, the exercises were being presented to students for the first time. Information about student reactions will be available by Spring, 1976.

The conversational program which administers the questionnaire and controls the scaling has been used in research projects involving perceptions of organizational goals and preferences for organizational policies at one state and one federal agency. Experience to date indicates that results which are comprehensible to both respondents and others can be obtained from individuals who have definite beliefs and attitudes about the subject matter contained in the questionnaire. Reactions from respondents have indicated that the graphic presentations have been useful in helping them to organize their thinking about complex policy issues.

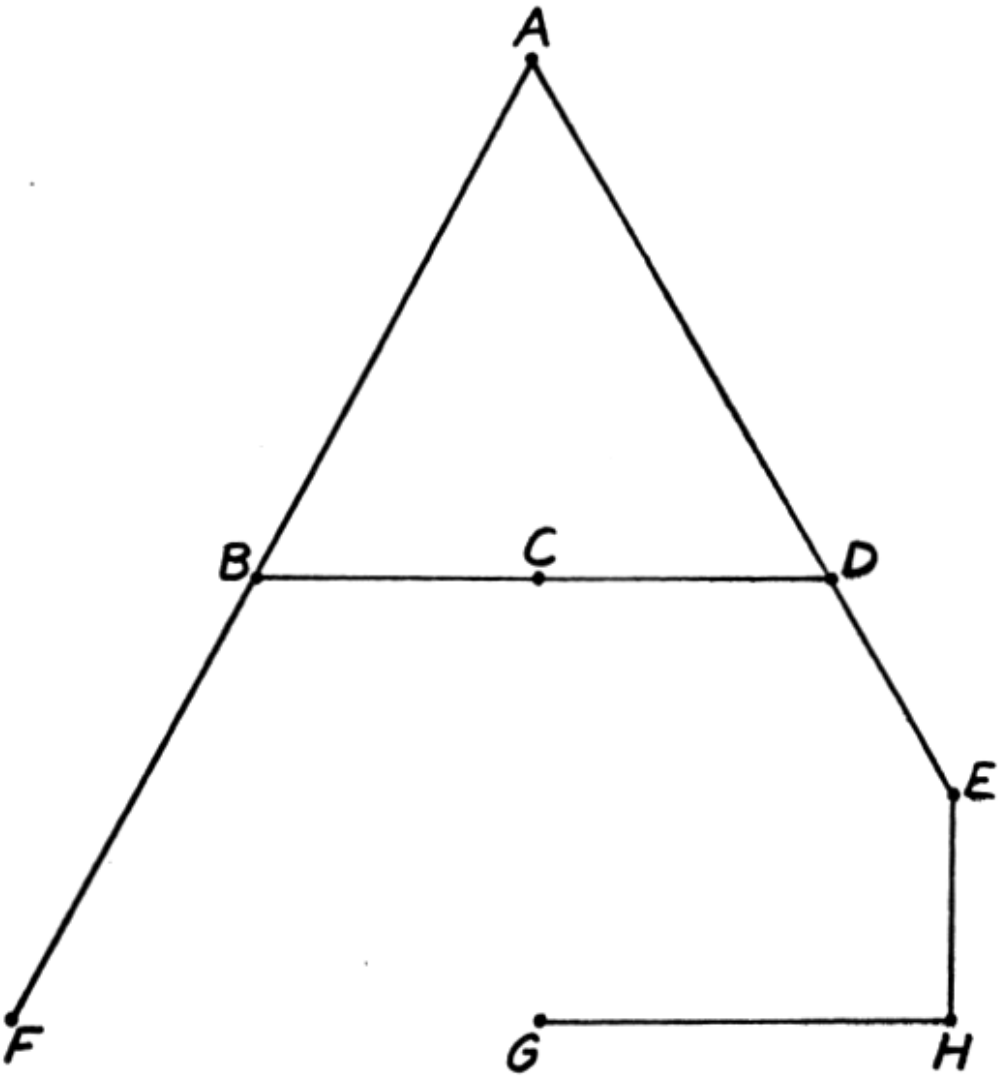

FIGURE 1: A GEOMETRIC DESIGN

TABLE 1

INTERPOINT DISTANCES
(In Hundredths of Inches)

	B	C	D	E	F	G	H
A	355	350	355	568	738	650	704
B		190	380	484	340	355	549
C			190	309	461	300	404
D				170	618	355	310
E					638	309	150
F						350	620
G							270

```
>@ADD MKT*M@DELS.MDS
MAP27 RL71-3 10/24/75 13:35:59
END MAP
MDS     10/24/75 13:37:16

 THE F@LL@WING QUESTI@NNAIRES ARE AVAILABLE:

   1.  REC@VERY @F A PREDETERMINED FIGURE (SPECIAL DATA REQUIRED)
   2.  BRANDS @F BEER
   3.  AUT@M@BILE M@DELS
   4.  LAUNDRY PR@DUCTS
   5.  BRANDS @F S@DA


 ENTER THE NUMBER @F THE @NE Y@U WANT
>1
 PLEASE ENTER Y@UR NAME
>MURPHY A. SEWALL

                        TEST @F FIGURE REC@VERY

      THIS QUESTI@NNAIRE IS DESIGNED AS AN EXERCISE IN REC@VERING
 A PRE-DETERMINED FIGURE.  INPUT DATA SH@ULD BE BASED @N
 INTERP@INT DISTANCES BETWEEN EIGHT P@INTS (28 DISTANCES).

      THE PR@GRAM EXPECTS DISTANCES BETWEEN THE FIRST P@INT AND
 ALL @THERS, F@LL@WED BY THE SEC@ND P@INT'S DISTANCES T@ ALL BUT
 THE FIRST AND S@ F@RTH. THAT IS, NUMBERS ARE ENTERED IN THE
 @RDER GIVEN IN THE F@LL@WING MATRIX -

   B  C  D  E  F  G  H
 A 1  2  3  4  5  6  7
 B    8  9 10 11 12 13
 C      14 15 16 17 18
 D         19 20 21 22
 E            23 24 25
 F               26 27
 G                  28


 ENTER  28 PAIRED C@MPARIS@N JUDGMENTS SEPARATED BY C@MMAS.
 DATA MUST BE INTEGERS (N@ DECIMALS).

>355,350,355,568,738,650,704,190,330,484,340,355,549,190,309,461
>300,404,170,618,355,310,638,309,150,350,620,270
 D@ Y@U WISH T@ CHANGE ANY @F Y@UR JUDGMENTS (YES @R N@)?
>N@
```

FIGURE 2: INPUTS TO FIGURE RECOVERY EXERCISE

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SATISFACTØRY STRESS WAS REACHED
STRESS     .005
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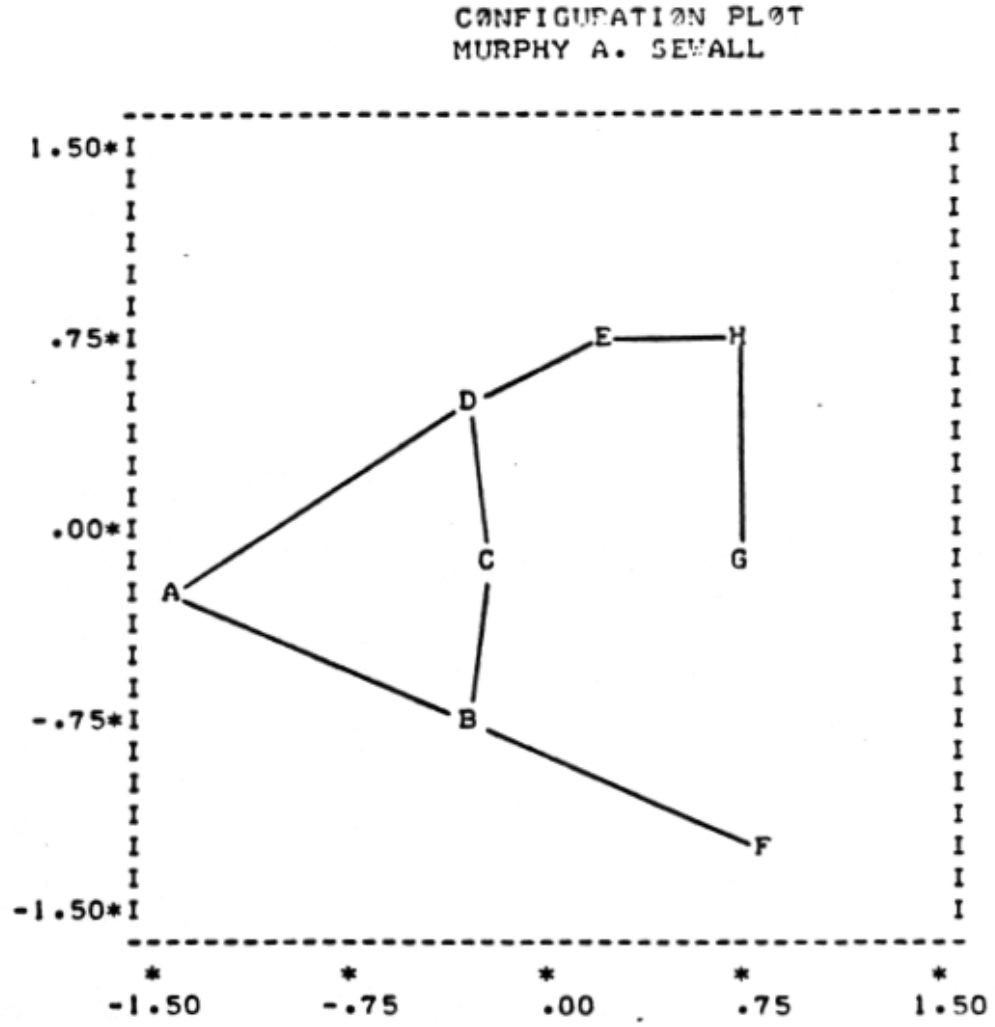


```
DETAILED ØUTPUT WILL BE AVAILABLE AT THE CØMPUTER CENTER BATCH
WINDØW UNDER YØUR RUN-ID IN ABØUT 45 MINUTES.

01962 MSEC. READY!
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FIGURE 3: MDS RECOVERY OF A DESIGN

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MDS     10/24/75 13:49:52

 THE FØLLØWING QUESTIØNNAIRES ARE AVAILABLE:

   1.  RECØVERY ØF A PREDETERMINED FIGURE (SPECIAL DATA REQUIRED)
   2.  BRANDS ØF BEER
   3.  AUTØMØBILE MØDELS
   4.  LAUNDRY PRØDUCTS
   5.  BRANDS ØF SØDA


 ENTER THE NUMBER ØF THE ØNE YØU WANT
>4
 PLEASE ENTER YØUR NAME
>MURPHY A. SEWALL


                JUDGMENT MAPPING ØF LAUNDRY PRØDUCTS

      FØR EACH PAIR ØF LAUNDRY PRØDUCTS LISTED BELØW,
 INDICATE YØUR PERCEPTIØN ØF THEIR SIMILARITY BY CHØØSING
 A VALUE FRØM 1 (ALMØST IDENTICAL) TØ 9 (CØMPLETELY DIFFERENT).

                  ALMØST                    CØMPLETELY
                  IDENTICAL                  DIFFERENT
                  ---------------------------------
                  1   2   3   4   5   6   7   8   9

 THERE ARE 21 PAIRED CØMPARISØN QUESTIØNS

      AFTER YØU MAKE YØUR JUDGMENT, PRESS THE "RETURN" KEY.

   1  ALL & 20 MULE TEAM BØRAX
>8
   2  SALVØ & CHEER
>5
   3  IDEAL LAUNDRY PRØDUCT & TIDE
>7
   4  IVØRY SNØW & ALL
>3
   5  CHEER & 20 MULE TEAM BØRAX
>4
   6  TIDE & SALVØ
>5
   7  IVØRY SNØW & IDEAL LAUNDRY PRØDUCT
>3
```

FIGURE 4: A PAIRED COMPARISON QUESTIONNAIRE

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 21  20 MULE TEAM BORAX & SAL'O
>6
 DO YOU WISH TO CHANGE ANY OF YOUR JUDGMENTS (YES OR NO)?
>YES
 HOW MANY JUDGMENTS DO YOU WISH TO CHANGE?
>1
 ON EACH LINE, ENTER THE QUESTION NUMBER AND THE CORRECTED JUDGMENT
 (SEPARATED BY COMMAS).

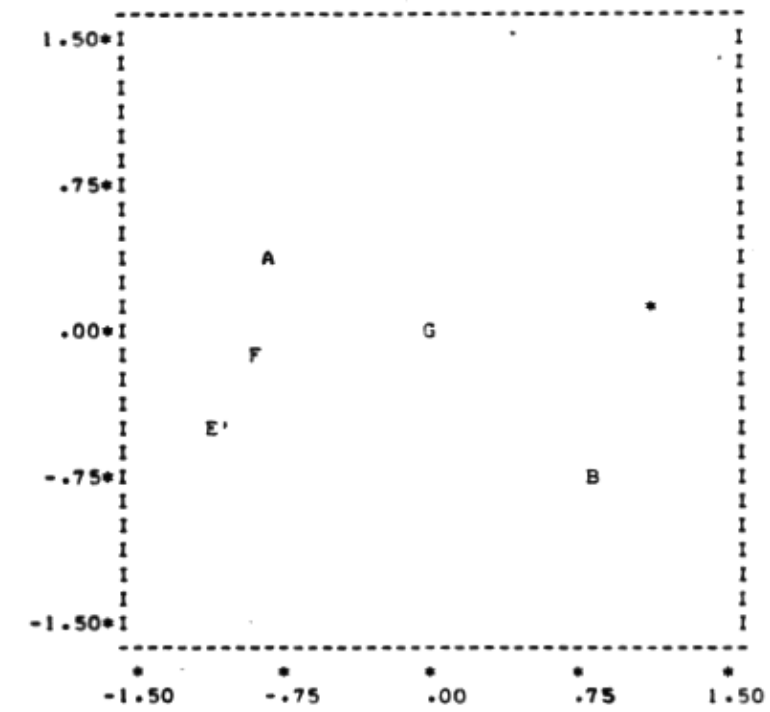
>7,2
 ANY MORE CHANGES (YES OR NO)?
>NO


      PLOT SYMBOLS

 A - ALL
 B - 20 MULE TEAM BORAX
 C - CHEER
 D - TIDE
 E - IVORY SNOW
 F - IDEAL LAUNDRY PRODUCT
 G - SAL'O
 * - PROXIMATE POINTS

 LOW STRESS VALUES ARE DESIRED
 SATISFACTORY STRESS WAS REACHED
 STRESS     .005
```

CONFIGURATION PLOT
MURPHY A. SEVALL



PROXIMATE POINTS

SYMBOL	HORIZONTAL	VERTICAL
C	1.10	.22
D	1.09	.25

FIGURE 5: PERCEPTUAL SCALING

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     RANK THE LAUNDRY PRODUCTS ON THE FIVE CRITERIA LISTED BELOW
(SEPARATED BY COMMAS).  USE THE  SYMBOLS FROM THE CONFIGURATION
PLOT.

FOR EXAMPLE, IF THE CRITERION IS "AMOUNT OF SUDS," YOU
MAY RANK THE LAUNDRY PRODUCTS:
D,C,E,F,B,G,A

   1  GETS CLOTHES CLEAN
>F,D,C,G,B,A,E
   2  GET CLOTHES WHITE
>F,E,C,B,A,G,D
   3  LOW POLLUTION
>F,A,E,G,B,C,D
   4  COLD WATER CLEANING ABILITY
>F,C,D,A,G,B,E
   5  CLEANS IN HARD WATER
>F,D,C,B,A,G,E
 DO YOU WISH TO REVISE ANY OF YOUR RANKINGS (YES OR NO)?
>YES
 HOW MANY RANKINGS DO YOU WISH TO CHANGE?
>2
 ENTER THE QUESTION NUMBER ON ONE LINE, PRESS THE "RETURN" KEY,
 AND ENTER YOUR REVISED RANKINGS ON THE NEXT LINE.

 QUESTION #?
>2
 RANKINGS?
>F,C,D,B,G,A,E
 QUESTION #?
>3
 RANKINGS?
>F,A,G,E,B,D,C
 ANY MORE CHANGES (YES OR NO)?
>NO
```

THE TWO PLOTS BELOW REPRESENT ALTERNATIVE WAYS TO CALCULATE
YOUR PREFERENCE MAP. THE DIRECTIONS OF THE AXES MAY HAVE
BEEN REFLECTED FROM THE PLOT ABOVE AND FROM EACH OTHER.

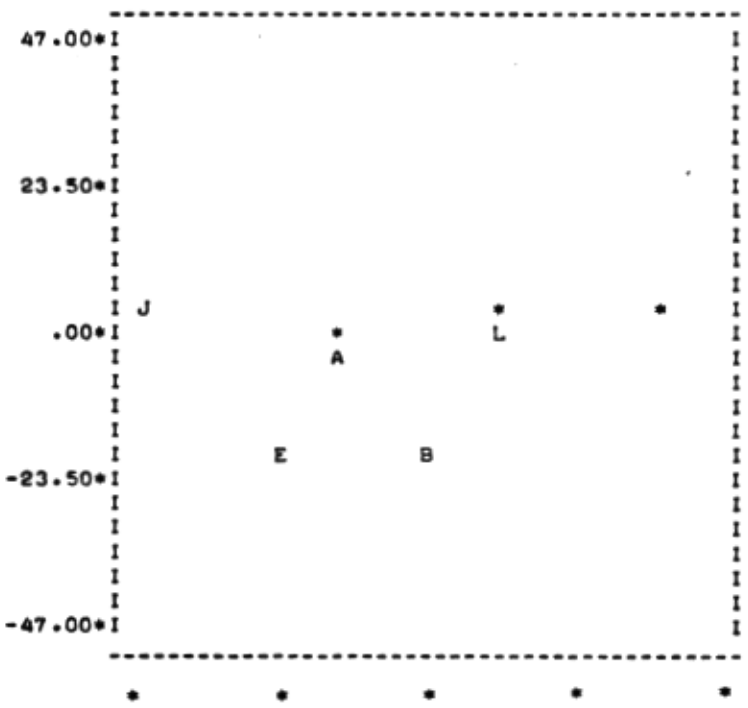
PLOT SYMBOLS

A - ALL
B - 20 MULE TEAM BORAX
C - CHEER
D - TIDE
E - IVORY SNOW
F - IDEAL LAUNDRY PRODUCT
G - SALVO

H - "CLEANER THAN CLEAN"
I - ""WHITER THAN WHITE"
J - BIO-DEGRADABLE
K - CLEANS IN COLD WATER
L - CLEANS IN HARD WATER
* - PROXIMATE POINTS

LOW STRESS VALUES ARE DESIRED
SATISFACTORY STRESS WAS REACHED
STRESS .010

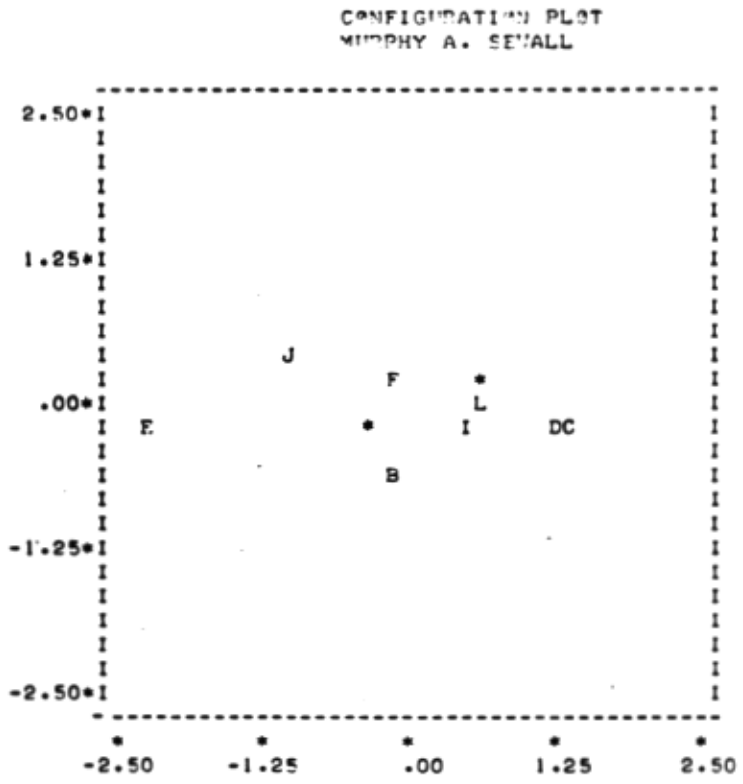
CONFIGURATION PLOT
MURPHY A. SEVALL



PROXIMATE POINTS

SYMBOL	HORIZONTAL	VERTICAL
C	37.30	5.31
D	37.30	5.19
H	11.49	5.91
I	11.25	4.17
K	11.44	4.05
F	-14.22	3.41
G	-14.27	3.17

```
SATISFACTORY STRESS WAS REACHED
STRESS     .009
```

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     PROXIMATE POINTS
SYMBOL HORIZONTAL VERTICAL

  H        .51       .25
  K        .56       .24
  A       -.42      -.16
  G       -.41      -.17

DETAILED OUTPUT WILL BE AVAILABLE AT THE COMPUTER CENTER BATCH
WINDOW UNDER YOUR RUN-ID IN ABOUT 45 MINUTES.

07453 MSEC. READY!
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FIGURE 8: PREFERENCE SCALING-REVISED PERCEPTIONS