

# THREE-ATTRIBUTE INTERRELATIONSHIPS FOR INDUSTRY-LEVEL DEMAND EQUATIONS

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## ABSTRACT

*In business simulations, industry-level product demand is typically determined by a variety of factors. In the marketplace, these variables are not independent, yet many simulation algorithms in the literature assume that they are. The multiple-market-segment industry-level demand equation detailed in this paper allows for correlations among three attributes that form the domain of the demand equation.*

## INTRODUCTION

Over twenty years ago, Gold and Pray (1983) presented the now frequently cited industry-level demand equation: (see equation 1)

where **P** is price, **M** is marketing and promotion effort, and **R** is the research (and development) effort for the particular product. The latter variable is included as a proxy to reflect product quality. Equation 1 was recently recommended in Gold (2003) as part of his systems-dynamics based approach. However, the interrelationships among price, promotion and research are not explicitly addressed by this equation. Furthermore, in Perotti and Pray's award winning paper (2000), they indicated that this model is unstable and does not exhibit desirable characteristics when using input variables in moderately extended ranges. This is clearly visible in Figure 1, which displays a surface map of this equation for two inputs (price and promotion):

To prevent the simulation players from straying into the domain region where this problem occurs, the input options available must be bounded. Additionally, the Gold and Pray model does not control for relationships between the primary demand generating variables: A joint distribution

$$Q(P, M, R) = g_1 P^{-(g_2+g_3P)} M^{(g_4+g_5M)} R^{(g_6+g_7R)} \quad (1)$$

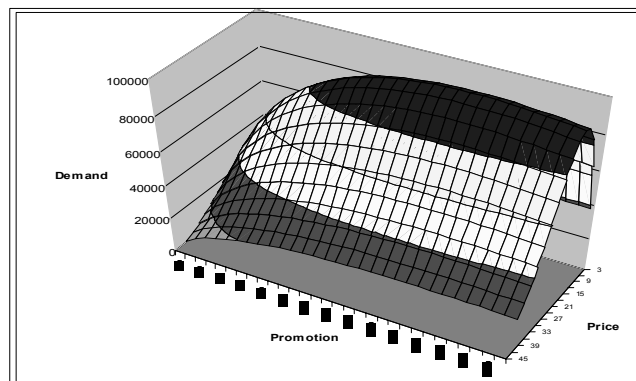


Figure 1: Surface map of Gold and Pray's industry-level demand equation for price and promotion inputs (Price scale reversed for readability)

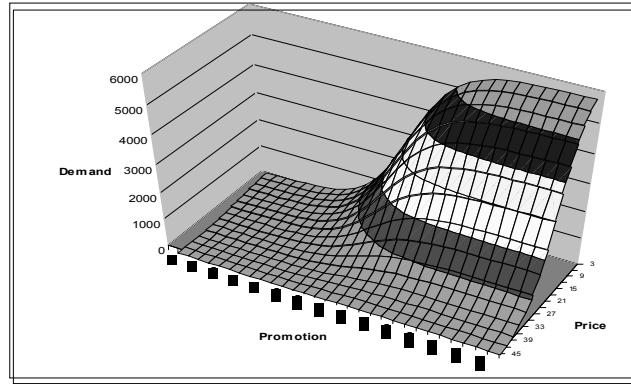


Figure 2: Surface map of Carvalho’s industry-level demand equation for price and promotion inputs (Price scale reversed for readability)

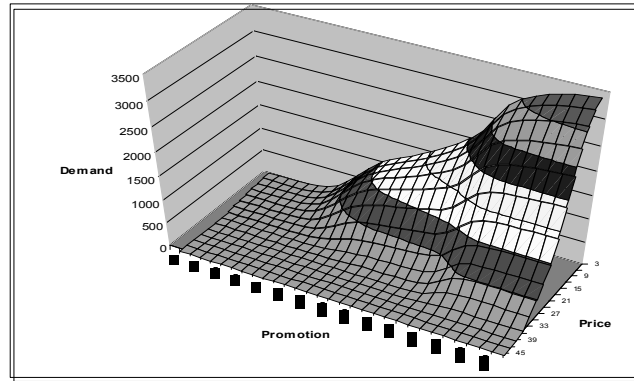


Figure 3: Surface map of Murff et al.’s multi-segment industry-level demand equation for price and promotion inputs (Price scale reversed for readability)

$$Q(P, M, R) = k(1 + e^{(P - \mu_P)/\sigma_P})^{-1}(1 + e^{-(M - \mu_M)/\sigma_M})^{-1} \quad (2)$$

$$Q(P, M) = k(-\sinh^{-1}((P - \mu_P)/\sigma_P) + g_P)(\sinh^{-1}((M - \mu_M)/\sigma_M) + g_M) \quad (3)$$

function has the form  $F_{X,Y}(x,y) = F_X(x)F_Y(y)$  if and only if the variables X and Y are independent (Grimmett & Stirzaker, 1992, p99). Thus, promotional expenditures have the same effect upon demand regardless of the product’s price or research effort. Suppose research effort is set at a fixed amount. If the product had a price of \$10.00 and promotional expenditures were doubled, demand would increase by a certain proportion. But, if the product was priced at \$30.00 and the promotional expenditure was doubled, the demand would increase by exactly the same proportion.

### BACKGROUND

Carvalho (1991) led the way for the use of probability in industry-level demand equations by demonstrating that the proportion of purchase distribution is actually a cumulative distribution function. As shown in Figure 2, for two input variables this approach resolved the monotonicity problems inherent in Gold and Pray’s equation and does not require artificial constraint of the domain. However, the interrelationship between price and promotion was not

explicitly included as Carvalho used independent logistic distributions: (See equation 2)

In a working-paper, Teach and Schwartz (2001) attempted to include domain interrelationships in a cumulative distribution based equation. They began with appropriately shifted independent inverse hyperbolic sine functions: (See equation 3)

They then applied non-orthogonal axes to introduce the dependencies in the domain variables. This distortion of the axes was quite cumbersome and extremely difficult to visualize. But, it is just a short step from equation 3 to one in which the correlation between two domain variables can be explicitly included without distorting the axes.

First, consider Carvalho’s use of the logistic distribution in equation 2. This function differs from the normal distribution primarily in its larger kurtosis. Next, consider Teach and Schwartz’s use of the inverse hyperbolic sine function. Equation 3 is related to the inverse tangent function, which is the basis of the Cauchy cumulative distribution. The Cauchy distribution is a special case of a Student’s t distribution with one degree of freedom. Finally, Student’s t distribution with infinite degrees of

$$Q(P, M) = Q_{\max} \int_{x=P}^{\infty} \int_{y=-\infty}^M \frac{1}{2\pi\sigma_P\sigma_M\sqrt{1-\rho^2}} e^{\left\{ -\frac{1}{2(1-\rho^2)} \left[ \left( \frac{x-\mu_P}{\sigma_P} \right)^2 + \left( \frac{y-\mu_M}{\sigma_M} \right)^2 - 2\rho \left( \frac{x-\mu_P}{\sigma_P} \right) \left( \frac{y-\mu_M}{\sigma_M} \right) \right] \right\}} dydx \quad (4)$$

$$f(x_P, x_M, x_R) = \frac{1}{(2\pi)^{3/2} (\det \Sigma)^{1/2}} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)} \quad (5)$$

$$\rho_{PR}^2 + \rho_{MR}^2 + \rho_{PM}^2 < 1 + 2\rho_{MR}\rho_{PM}\rho_{PR} \quad (6)$$

freedom is the normal distribution. This reasoning led Murff et al. (2005) to adapt the bivariate normal distribution to create an industry-level demand equation:(see equation 4) Price and promotion are now explicitly interrelated through the correlation ( $\rho$ ). Furthermore, these authors suggested that the summation of several equations of this form would allow for multiple market segments. Different groups of buyers have different responses to product attributes (Rogers, 1962). For example, “innovators” are risk-takers driven by the thrill of discovery. They are a small segment (2.5%) willing to pay higher prices and are less responsive to large promotion efforts as they have multiple information sources. The “late majority” is a large segment (34%) preferring the tried-and-true, with lower prices and larger promotion efforts required. This equation allows for the presence of both groups simultaneously. As seen in Figure 3, this summation still displays the desired monotonicity characteristics, but equation 4 is limited to only two input variables.

### THIS PAPER

This paper extends the adapted bivariate normal model presented in Murff et al. (2005) to an adapted multivariate normal model to allow for interrelationships among three attributes. In the theoretical discussion, matrices were used for brevity. Inequality 6 is the key result as it defines the relationship required among the three correlations. In the practical discussion, one method for developing appropriate parameters is considered. In the algorithm section, a quick computational method for incorporating this model into the “black box” is provided. The discussion just before the conclusion explains the authors’ motivation for developing this model based on experiences in the administration of business simulations.

### THEORETICAL DISCUSSION

The multivariate normal probability density function for a three-dimensional domain will be the probability of purchase function, defined by the equation (Johnson & Wichern, 1992, p128): (See equation 4)

where:

$$\mathbf{X} = \begin{bmatrix} x_P \\ x_M \\ x_R \end{bmatrix}, \mu = \begin{bmatrix} \mu_P \\ \mu_M \\ \mu_R \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} \Sigma_{PP} & \Sigma_{PM} & \Sigma_{PR} \\ \Sigma_{PM} & \Sigma_{MM} & \Sigma_{MR} \\ \Sigma_{PR} & \Sigma_{MR} & \Sigma_{RR} \end{bmatrix}$$

and  $\Sigma_{ii}$  denotes the variance of variable  $i$  and  $\Sigma_{ij}$  denotes the covariance between variables  $i$  and  $j$ . The difficulty in using equation 5 lies entirely in the development of an appropriate covariance matrix  $\Sigma$  as it must be positive definite to force the determinant of  $\Sigma$  to be positive (Watkins, 2002, p49).

The correlation matrix is related to the covariance matrix  $\Sigma$  as follows (Johnson & Wichern, 1992, p59):

$$\Sigma = \begin{bmatrix} \Sigma_{PP} & \Sigma_{PM} & \Sigma_{PR} \\ \Sigma_{PM} & \Sigma_{MM} & \Sigma_{MR} \\ \Sigma_{PR} & \Sigma_{MR} & \Sigma_{RR} \end{bmatrix} = \begin{bmatrix} \sigma_P & 0 & 0 \\ 0 & \sigma_M & 0 \\ 0 & 0 & \sigma_R \end{bmatrix} \begin{bmatrix} 1 & \rho_{PM} & \rho_{PR} \\ \rho_{PM} & 1 & \rho_{MR} \\ \rho_{PR} & \rho_{MR} & 1 \end{bmatrix} \begin{bmatrix} \sigma_P & 0 & 0 \\ 0 & \sigma_M & 0 \\ 0 & 0 & \sigma_R \end{bmatrix}$$

where  $\sigma_P$ ,  $\sigma_M$  and  $\sigma_R$  are the standard deviations of price, promotion and research respectively and  $\rho_{PM}$ ,  $\rho_{PR}$  and  $\rho_{MR}$  are the correlations between price and promotion, price and research, and promotion and research respectively.

If the correlation matrix is known to be positive definite,  $\Sigma$  also must be positive definite (Watkins, 2002, p47). Thus, developing an appropriate correlation matrix will result in an appropriate covariance matrix. This may be done through the use of the Cholesky Decomposition Theorem (Watkins, 2002, p34) which states that a matrix is positive definite if and only if it can be decomposed into a unique upper triangular matrix  $R$  with positive main diagonal entries such that:

$$\begin{bmatrix} 1 & \rho_{PM} & \rho_{PR} \\ \rho_{PM} & 1 & \rho_{MR} \\ \rho_{PR} & \rho_{MR} & 1 \end{bmatrix} = \begin{bmatrix} r_{PP} & 0 & 0 \\ r_{PM} & r_{MM} & 0 \\ r_{PR} & r_{MR} & r_{RR} \end{bmatrix} \begin{bmatrix} r_{PP} & r_{PM} & r_{PR} \\ 0 & r_{MM} & r_{MR} \\ 0 & 0 & r_{RR} \end{bmatrix} = \mathbf{R}^T \mathbf{R}$$

As the main diagonal entries of  $R$  must be positive, a relationship for the three correlations results when the matrix multiplication is carried out: (see equation 6)

If inequality 6 holds, the correlation matrix is positive definite which results in a positive definite covariance matrix which allows for an appropriate proportion of purchase function which yields an appropriate industry-level demand function.

Unfortunately, no closed form exists for the multivariate normal cumulative distribution function, thus equation 5 must be integrated, remembering to reverse the direction of integration for price as demand falls as price increases (Gold and Pray, 1990). Let  $Q_{\max}$  be the highest possible industry-level demand for the market segment defined by  $\mu$  and  $\Sigma$ . Then the industry-level demand equation for this segment is: (see equation 7)

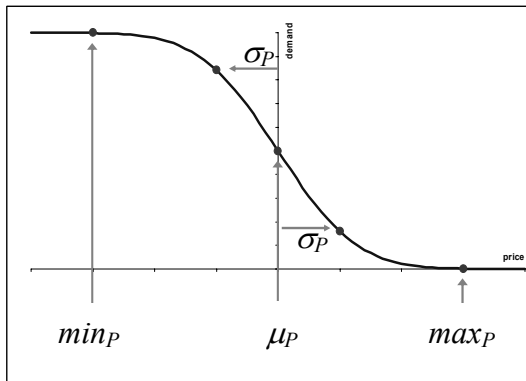


Figure 4a: Demand as a function of price

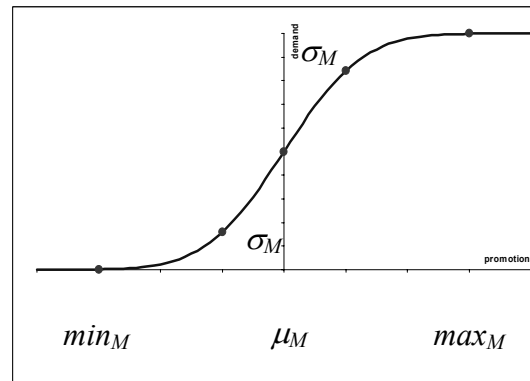


Figure 4b: Demand as a function of promotion

$$Q(\mathbf{p}, \mathbf{m}, \mathbf{r} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}, \mathbf{Q}_{\max}) = Q_{\max} \int_{-\infty}^{\mathbf{p}} \int_{-\infty}^{\mathbf{m}} \int_{-\infty}^{\mathbf{r}} \mathbf{f}(\mathbf{x}_p, \mathbf{x}_M, \mathbf{x}_R) d\mathbf{x}_p d\mathbf{x}_M d\mathbf{x}_R \quad (7)$$

$$Q(\mathbf{p}, \mathbf{m}, \mathbf{r}) = \sum_{i=1}^n Q(\mathbf{p}, \mathbf{m}, \mathbf{r} \mid \boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i, \mathbf{Q}_{\max, i}) \quad (8)$$

If  $n$  market segments are to be included in the simulation, let  $\mu_i$ ,  $\Sigma_i$  and  $Q_{\max, i}$  be the set of constants used to define industry-level demand for market segment  $i$ . The overall industry-level demand function may be found by: (see equation 8)

### PRACTICAL DISCUSSION

The constants may be easily adjusted by the game administrator as they have practical interpretations. Furthermore, the evolution of a particular market segment may now be readily modeled by simply changing the appropriate constants during the game play itself. For example,  $\mu_p$ ,  $\mu_M$  and  $\mu_R$  are the points of diminishing returns (that is, the inflection points of the demand curve) for price, promotion and research, respectively.  $\sigma_p$  is the distance from  $\mu_p$  where the marginal impact on demand with respect to changes in price inflects.  $\sigma_M$  and  $\sigma_R$  are defined similarly. Figures 4a and 4b provide visualization for these constants. Demand as a function of research has not been included as the function is identical to that of figure 4b with research replacing promotion. Small values of  $\sigma_i$  result in a demand that is highly sensitive to changes in variable  $i$  near  $\mu_i$ . Large values of  $\sigma_i$  would result in a demand that is less sensitive near  $\mu_i$ . One method for estimating these constants involves identifying the minimum and maximum values where a response to a change is seen. Then,  $\mu_i = (\max_i + \min_i) / 2$  and  $\sigma_i = (\max_i - \min_i) / 6$ .

Correlations near +1 indicate strong positive interrelationships. For example, consider a situation where promotion and research are strongly and positively correlated. This means that an increase in research will increase the impact in promotion, and vice versa. This would represent a rapidly changing technology industry and extensive R&D would be needed to keep up with the

competition and high advertising expenditures would be needed to keep the potential customers informed about all the product changes taking place. Cell phones, computer games and digital organizers are products that might encounter this condition. A correlation near -1 indicates a strong negative interrelationship, that is, an increase in one of the variables will decrease the impact of the other, and vice versa. Variables not influencing the impacts of others will have correlations near 0. Appendix A provides valid ranges on a  $\Delta=0.1$  grid for the correlation between price and research given specific values for the correlation between price and promotion and the correlation between promotion and research.

To simplify this process as much as possible, the game developers should provide to the game administrator a selection of preset choices along with a verbal description of their practical meaning in the marketplace. This will place the complexity of the process in the “black box” and allow the game administrator to focus on the lesson to be taught.

### ALGORITHM

For example, consider the two market segments defined by the constants in Table 1. Let the input values be  $\mathbf{X}_p = 25$ ,  $\mathbf{X}_M = 1350$ , and  $\mathbf{X}_R = 210$ . Next, check that a valid set of correlations has been chosen by consulting Appendix A. If an invalid set has been chosen, an adjustment in one of the correlations will be needed before continuing. The calculation algorithm that follows is then applied to generate a value for overall industry-level demand.

This procedure was adapted from the multivariate normal cumulative distribution function algorithm developed by Genz (1992). The notation **normcdf**(Z) refers to the left-tailed probability associated with the standard normal cumulative distribution function at Z. The notation **norminv** refers to the inverse of **normcdf**. The Excel

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workbook developed to perform the calculations shown below is available from the first author.

### DISCUSSION

Many of the industry-level demand models used in simulations have a variety of flaws (Wolfe and Teach, 1987). Most of these flaws are not immediately detectable by participants or game administrators. However, these flaws cause subtle but important and detrimental changes in the nature of the competition, which affect game participant understanding of the results of each round of play. For example, some games create elasticities that are greater at the industry level than at the firm level. This causes participants to encounter unrealistic behavior in the game's reaction to the changes in strategies and decisions. The algorithm described in this paper then was developed to correct flaws in existing industry-level demand algorithms.

Further, many of the demand algorithms have very restrictive operating ranges. The algorithms work well if the game participants do not enter extreme values as decisions. While logic suggests that these extreme values do not make economic sense, they do occur, especially when teams make last ditch efforts to either break the game or win by making unconventional decisions. One of the authors of this paper had a simulation team submit a \$1 million price on a product that had a normal price range of between \$20 and \$25. When confronted, the team members said they knew the game had a feature that would reduce demand as prices increased, but they hoped the algorithm would not allow demand to fall to zero. Therefore, they believed the sale of only a few products would generate a large profit. This use of illogical decisions generally occurs only in the last round of a game when a team's firm is lagging behind in performance. These players are hoping for an error in mathematical or programming logic that will save their firm. The algorithm presented here allows for appropriate responses to unconventional decisions without restricting the domains of the input variables.

When games have been used repeatedly in academic settings, institutional memory can develop. Students will often seek out those who used the simulation in prior semesters to obtain the way to "win the game." This then detracts from the lesson being taught and reduces the simulation to a simple competition among the players. By simply altering the parameters defining the market segments involved in the simulation, the strategy of blindly following the choices of those who did well in previous semesters is no longer viable. The focus of the simulation can then return to the lesson to be learned. The algorithm developed in this paper handles these changes with ease.

### CONCLUSIONS

In the actual marketplace, demand-generating decision variables are known to have interactions (Arora, 1979). The

model presented in this paper incorporates these important interactions. As an example, a higher than average price may be offset by more aggressive advertising expenditures, whereas, at a lower than average price, the additional advertising expenditures might not generate the same effect on demand. Without the ability to model interactions that affect decisions and strategies, the understanding and learning that takes place during business simulations may be misleading and could potentially teach the wrong lessons.

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## Appendix A

Valid ranges (min top, max bottom) on a  $\Delta=0.1$  grid for  $\rho_{PR}$  given specific values of  $\rho_{PM}$  and  $\rho_{MR}$

		Correlation between price and promotion, $\rho_{PM}$																		
		-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
Correlation between promotion and research, $\rho_{MR}$	-0.9	0.7*	0.5	0.4	0.2	0.1	0.1	-0.1	-0.2	-0.3	-0.5	-0.6	-0.6	-0.7	-0.8	-0.8	-0.9	-0.9	-0.9	
	0.9	0.9	0.9	0.9	0.8	0.8	0.7	0.6	0.6	0.5	0.3	0.2	0.1	-0.1	-0.1	-0.2	-0.4	-0.5	-0.7	
	-0.8	0.5	0.3	0.2	0.1	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9
	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	-0.1	-0.2	-0.3	-0.5	
	-0.7	0.4	0.2	0.1	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9
	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	-0.1	-0.2	-0.4	
	-0.6	0.2	0.1	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8
	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	-0.1	-0.2	
	-0.5	0.1	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8
	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	-0.1	
	-0.4	0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.7
	0.7	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.8	0.7	0.6	0.5	0.4	0.3	0.2	-0.1
	-0.3	-0.1	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.8	-0.6
	0.6	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.8	0.7	0.6	0.5	0.4	0.3	0.1	0.1
	-0.2	-0.2	-0.4	-0.5	-0.6	-0.7	-0.8	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.7	-0.6
	0.6	0.7	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.8	0.7	0.6	0.5	0.4	0.2	0.2
	-0.1	-0.3	-0.5	-0.6	-0.7	-0.8	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.7	-0.6	-0.5
0.5	0.6	0.7	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.8	0.7	0.6	0.5	0.3	0.3	
0.1	-0.5	-0.6	-0.7	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.8	-0.7	-0.6	-0.5	-0.3	-0.3	
0.3	0.5	0.6	0.7	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.7	0.6	0.5	0.5	
0.2	-0.6	-0.7	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.8	-0.7	-0.6	-0.5	-0.4	-0.2	-0.2	
0.2	0.4	0.5	0.6	0.7	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.7	0.6	
0.3	-0.6	-0.8	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.1	-0.1	
0.1	0.3	0.4	0.5	0.6	0.7	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.8	0.6	0.6	
0.4	-0.7	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	0.1	0.1	
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.7	0.7	
0.5	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.1	0.1	
0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.8	
0.6	-0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.1	0.2	0.2	
0.2	0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.8	
0.7	-0.9	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.1	0.2	0.4	0.4	
0.4	0.2	0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	
0.8	-0.9	-0.9	-0.9	-0.9	-0.9	-0.8	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.1	0.2	0.3	0.5	0.5	
0.5	0.3	0.2	0.1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	
0.9	-0.9	-0.9	-0.9	-0.8	-0.8	-0.7	-0.6	-0.6	-0.5	-0.3	-0.2	-0.1	0.1	0.1	0.2	0.4	0.5	0.7	0.7	
0.7	0.5	0.4	0.2	0.2	0.1	0.1	0.1	0.2	0.3	0.5	0.6	0.6	0.7	0.8	0.8	0.9	0.9	0.9	0.9	

\* This cell would read as “Correlations between price and research ranging between 0.7 and 0.9 are allowed when the correlation between price and promotion is -0.9 and the correlation between promotion and research is -0.9.”