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**SIMULATION STUDY OF STOCHASTIC CHANNEL
REDISTRIBUTION**

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ABSTRACT

In this paper, we investigate sales redistribution between direct (i.e., "on-line") and indirect channels when market demand exhibits a general stochastic pattern. We model the dynamics of the inter-channel diffusion by a system of two differential equations, and then obtain a simple analytical expression of stability conditions for channel redistribution when the market demand is deterministic. The stability conditions are unfortunately unattainable under more realistic market situations where the demand contains uncertain fluctuations. The results from an extensive simulation study indicate that the stability conditions obtained under deterministic market remain applicable to uncertain market situations, including the demand following a stochastic fluctuation and the diffusion parameters following a known distribution.

INTRODUCTION

The increasing success of electronic commerce has made disintermediation possible in many industries. Consumers now can purchase products either from direct channel or from indirect channel [Majumdar and Venkatram, 1995]. Dell proves the direct model is viable. In many cases it is the best strategy for vendors trying to cut distribution costs. As a result, many manufacturers incorporated direct channel into their distribution channels. For example, HP sold its small and midsize business line of products on its web site, and IBM also rolled out its direct sales web site to its largest accounts [Schwartz and Briody, 1999]. These direct channels exist in parallel with the conventional indirect channels [Sridhar, 1998].

However, the existence of both direct channel and indirect channel arises many interesting questions. For example, does adding additional channel mean more sales? How much should each channel capacity be? How to redistribute channels among distribution channels? In this paper, we will study the channel redistribution on direct and indirect channels. Mainly we will study what effect the different market sizes have on the channel redistribution.

In this paper, we will first develop a two-dimensional diffusion model, which mainly consists of a

system of two differential equations. We also obtain key results regarding the steady-state redistribution when market size is deterministic. Then, we will study the demand redistribution between direct and indirect channels under stochastic market size. We will choose two common patterns for market size, namely exogenous random disturbance, and stochastic aggregate fluctuation, to study the demand allocation on direct and indirect channels. Since it is difficult to get a closed-form solution when market size is stochastic, we will use simulation to study the effect of market size on channel redistribution. Finally we will assume parameters in differential equations are stochastic, and use simulation to study the demand distribution on two channels.

MODEL

Consider a simple supply-chain system in an established market, in which a firm is faced with a decision to adopt a direct channel. Suppose the total market size at time t is given as $N(t) > 0$, which is assumed to be continuous and bounded ($N(t) \leq K < \infty$). The primary issue to be examined before a decision on the direct channel can be made pertains to how market sales would redistribute upon the amendment of a direct channel.

Upon the addition of a direct channel, there will be demand diffusion between the two channels. For example, the rate of change in sales via direct channel (or indirect channel), denoted as $\dot{x}_1(t)$ ($\dot{x}_2(t)$), will be affected by negative word-of-mouth (WOM) effect ξ_1 (ξ_2), positive sales promotion (PRO) effect η_1 (η_2), and loyal preference u (v). The loyal preference u (v) is measured in choice probability for direct (indirect) channel by a loyal customer of the firm.

In the most recent research by Yao and Liu (2003), the dynamics of the redistribution process is characterized by a system of ordinary differential equations extended from Bass (1969), and Muller (1983). Equation (1) is the ordinary differential equation of the system adopted in their paper. The notations are summarized below:

$N(t)$: the potential market size

$x_1(t)$: sales transacted via direct channel at time t .

- $x_2(t)$: sales transacted via indirect channel at time t .
 ξ_1 : diffusion coefficient for negative word-of-mouth moving away from direct channel.
 ξ_2 : diffusion coefficient for negative word-of-mouth moving away from indirect channel.
 η_1 : diffusion coefficient for promotion effect toward direct channel.
 η_2 : diffusion coefficient for promotion effect toward indirect channel.
 u : coefficient of loyal preference for direct channel.
 v : coefficient of loyal preference for indirect channel.

$$\begin{cases} \dot{x}_1(t) = -\xi_1 x_1(t) + \eta_1(N(t) - x_1(t) - x_2(t)) + ux_2(t) \\ \dot{x}_2(t) = -\xi_2 x_2(t) + \eta_2(N(t) - x_1(t) - x_2(t)) + vx_1(t) \end{cases} \quad (1)$$

If the market size $N(t)$ is deterministic, then the following proposition and theorem hold.

Proposition. *If $uv \leq \xi_1 \xi_2$, then there exists an equilibrium solution $\bar{x}(t)$ that has the following properties:*

1. *The equilibrium is feasible: $\bar{x}(t) \geq 0$, and $x_1(t) + x_2(t) \leq N(t)$*
2. *The equilibrium is proportionate to the total market:*

$$\bar{x}(t) = \begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \end{bmatrix} = \frac{N(t)}{\Delta} \begin{bmatrix} \bar{r}_1 \\ \bar{r}_1 \end{bmatrix}$$

where

$$\Delta = \bar{r}_1 + \bar{r}_2 + \xi_1 \xi_2 - uv$$

$$\bar{r}_1 = \xi_2 \eta_1 + u \eta_2$$

$$\bar{r}_2 = v \eta_1 + \xi_1 \eta_2$$

3. *The cross-channel redistribution ratio is time-invariant:*

$$\frac{\bar{x}_1(t)}{\bar{x}_2(t)} = \frac{\bar{r}_1}{\bar{r}_2} = \frac{\xi_2 \eta_1 + u \eta_2}{v \eta_1 + \xi_1 \eta_2}$$

Proof: see Yao & Liu (2003).

By proposition, the equilibrium redistribution, if it exists, will be solely dependent on the diffusion parameters, regardless of the initial conditions.

Theorem. *If $u \geq \eta_1$, $v \geq \eta_2$, and $\xi_2 \eta_1 + \xi_1 \eta_2 > 0$, and if the market is continuous and bounded (i.e., $N(t)$ is continuous in t , and $0 < N(t) \leq K$, where K is a positive constant), then the equilibrium redistribution $\bar{x}(t)$ is asymptotically stable.*

Proof: see Yao & Liu (2003).

A redistribution solution is said to be asymptotically stable if it is stable regardless of the initial status of the system. In other words, an asymptotically stable redistribution will ultimately settle down into a steady state regardless of when to add a direct channel and what volume of existing sales exist at the time of amendment of a direct channel. Intuitively, an equilibrium redistribution can be unstable in the sense that if a change or disturbance occurs to the current equilibrium situation, the system may move away from the equilibrium state and then diverge thereon, which is highly undesirable.

The stability conditions are unfortunately unattainable under more realistic market situations where the demand contains uncertain fluctuations and is auto-correlated. In next section, we will use simulation to study if the stability conditions obtained under constant market remain applicable to uncertain market situations, including the demand following a known distribution and stochastic diffusion parameters.

SIMULATION STUDY

In this section, we investigate, via simulation, on how different time-variant market size affects the channel redistribution. We simulate 1000 times for each case, and collect the sample means of demand on both channels when they are stable, also we will calculate 95% confidence interval (CI) for the redistribution ratio.

First we will assume market size is variable with known distribution, and we will consider two cases here, one is exogenous random disturbance which means the demand is deterministic with some disturbance (white noise); the other is stochastic aggregate fluctuation which means the fluctuation could be cumulative.

Case 1: Exogenous Random Disturbance, i.e.,

$$N(t) = N_0 + N_1(1 - e^{-ct}) + \varepsilon_t$$

Where ε_t is a white noise, and $\varepsilon_t \sim N(0, \sigma^2)$ with σ^2 known.

Example 1: we assume the parameters are as following:

$$\xi_1 = 0.03 \quad u = 0.004 \quad \eta_1 = 0.002$$

$$v = 0.01 \quad \xi_2 = 0.006 \quad \eta_2 = 0.005$$

$$N_0 = 1000 \quad N_1 = 2000 \quad \sigma = 100 \quad c = 0.3$$

Figure 3.1 gives the redistribution trajectories for this case. Both demands on direct and indirect channel increases as market size increases.

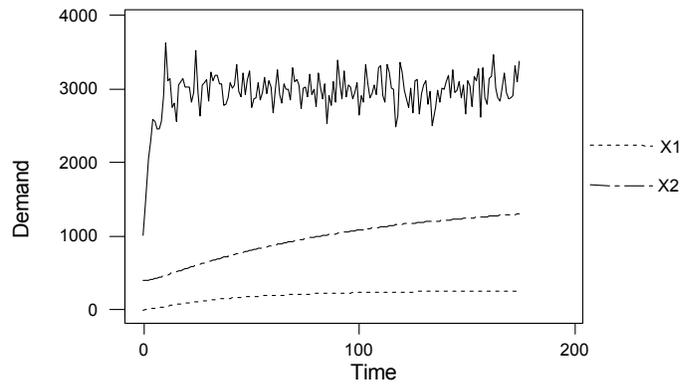


Figure 3.1 Increasing Redistribution Solutions

Table 3.1 summarizes the cross-channel redistribution ratio when the demands reach equilibrium under different

initial conditions. We find the demands on both channel are still stable regardless of the initial demand on both channels.

Initial condition	\bar{x}_1	\bar{x}_2	\bar{x}_1 / \bar{x}_2	95% CI
$x_1(0)=0, x_2(0)=50$	281	1492	0.1883	(0.1881, 0.1886)
$X_1(0)=0, x_2(0)=300$	281	1491	0.1883	(0.1874, 0.1884)
$X_1(0)=0, x_2(0)=800$	280	1492	0.1883	(0.1882, 0.1885)

Table 3.1 The cross-channel redistribution ration on both channels

Example 2:

$$\begin{aligned} \xi_1 &= 0.15 & u &= 0.004 & \eta_1 &= 0.005 \\ v &= 0.01 & \xi_2 &= 0.05 & \eta_2 &= 0.005 \\ N_0 &= 1000 & N_1 &= 2000 & \sigma &= 200 & c &= 0.3 \end{aligned}$$

This example shows the decreasing channel redistribution.

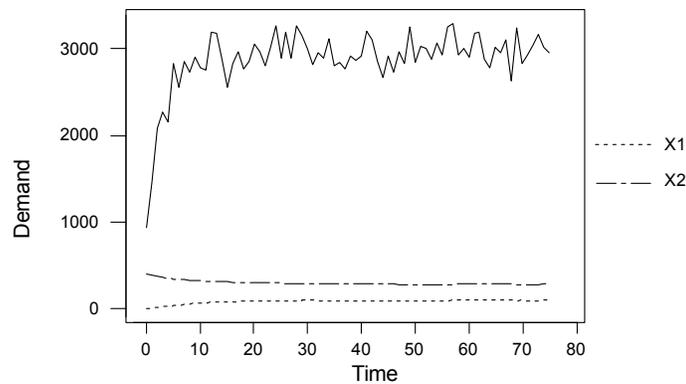


Figure 3.2 Decreasing Redistribution Solutions

Developments in Business Simulation and Experiential Learning, Volume 30, 2003

We list more simulation examples in table 3.2. All the simulation results show that the proposition and theorem still hold. It indicates the equilibrium redistribution exists and it is asymptotically stable as long as $u \geq \eta_1$, $v \geq \eta_2$

even if the market size follows exogenous random disturbance. This simulation results are consistent with Yao & Liu's (2003).

Parameters	\bar{x}_1	\bar{x}_2	\bar{x}_1 / \bar{x}_2	$(\xi_2 \eta_1 + u \eta_2) / (v \eta_1 + \xi_1 \eta_2)$
$\xi_1 = 0.09$ $u = 0.01$ $\eta_1 = 0.006$ $v = 0.05$ $\xi_2 = 0.06$ $\eta_2 = 0.01$ $\sigma = 100$ $c_2 = 0.1$	210.3	548.49	0.38339	0.3833
$\xi_1 = 0.3$ $u = 0.2$ $\eta_1 = 0.1$ $v = 0.1$ $\xi_2 = 0.3$ $\eta_2 = 0.009$ $\sigma = 100$ $c_2 = 0.1$	831.9	332.3	2.5038	2.5039
$\xi_1 = 0.1$ $u = 0.2$ $\eta_1 = 0.1$ $v = 0.08$ $\xi_2 = 0.3$ $\eta_2 = 0.03$ $\sigma = 100$ $c_2 = 0.3$	1767	540	3.27	3.27
$\xi_1 = 0.03$ $u = 0.004$ $\eta_1 = 0.002$ $v = 0.01$ $\xi_2 = 0.006$ $\eta_2 = 0.005$ $N_0 = 1000$ $N = 3000$ $\sigma = 100$ $c_2 = 0.3$	280	1489	0.188	0.188
$\xi_1 = 0.6$ $u = 0.004$ $\eta_1 = 0.005$ $v = 0.01$ $\xi_2 = 0.1$ $\eta_2 = 0.005$ $\sigma = 100$ $c_2 = 0.3$	24.5	143	0.17	0.17

Table 3.2 Simulation results of sample means for $N(t) = N_0 + N_1(1 - e^{-c_2 t}) + \varepsilon_t$

Case 2: Stochastic Aggregate Fluctuation, i.e.,

$$N(t) = N_0 + (N_1 + \varepsilon_t)(1 - e^{-c_2 t}).$$

Where $\varepsilon_t \sim N(0, \sigma^2)$ with σ^2 known.

Example 3: In this example, we assume

$$\xi_1 = 0.03 \quad u = 0.004 \quad \eta_1 = 0.002$$

$$v = 0.01 \quad \xi_2 = 0.006 \quad \eta_2 = 0.005$$

$$N_0 = 1000 \quad N_1 = 2000 \quad \sigma = 100 \quad c = 0.3$$

Figure 3.3 illustrates the increasing redistribution trajectory for this example.

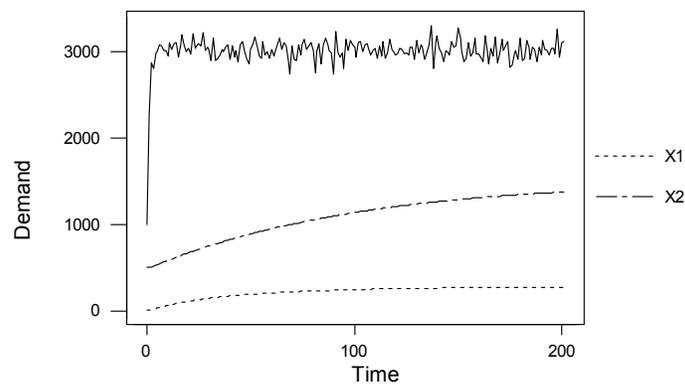


Figure 3.3 Increasing Redistribution Solutions

Example 4: In this example, we assume:

$$\begin{aligned} \xi_1 &= 0.15 & u &= 0.004 & \eta_1 &= 0.005 \\ v &= 0.01 & \xi_2 &= 0.05 & \eta_2 &= 0.005 \\ N_0 &= 1000 & N_1 &= 2000 & \sigma &= 200 & c &= 0.3 \end{aligned}$$

Figure 3.4 shows the decreasing trajectory solutions for this example.

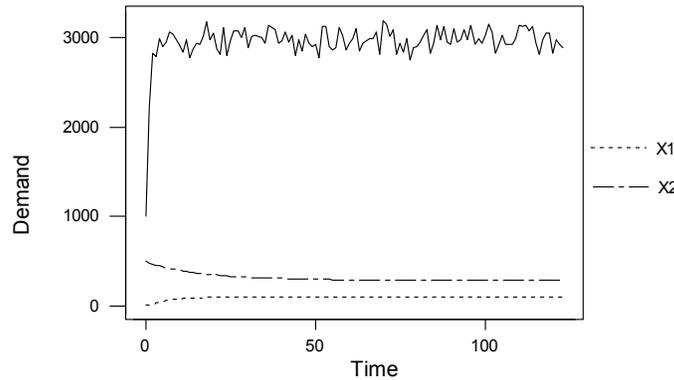


Figure 3.4 Decreasing Redistribution Solutions

We list more examples in table 3.3 for case 2. All the simulation results show that the proposition and theorem hold again in this case. It implies even if the market size follows stochastic aggregate fluctuation, the equilibrium

redistribution exists and it is asymptotically stable as long as $u \geq \eta_1, v \geq \eta_2$.

Parameters	\bar{x}_1	\bar{x}_2	\bar{x}_1 / \bar{x}_2	$(\xi_2 \eta_1 + u \eta_2) / (v \eta_1 + \xi_1 \eta_2)$
$\xi_1 = 0.09$ $u = 0.01$ $\eta_1 = 0.006$ $v = 0.05$ $\xi_2 = 0.06$ $\eta_2 = 0.01$ $\sigma = 100$ $c_2 = 0.1$	209	546	0.3832	0.3833
$\xi_1 = 0.3$ $u = 0.2$ $\eta_1 = 0.1$ $v = 0.1$ $\xi_2 = 0.3$ $\eta_2 = 0.009$ $\sigma = 100$ $c_2 = 0.1$	832	333	2.503	2.5039
$\xi_1 = 0.1$ $u = 0.2$ $\eta_1 = 0.1$ $v = 0.08$ $\xi_2 = 0.3$ $\eta_2 = 0.03$ $\sigma = 100$ $c_2 = 0.3$	1770	540	3.27	3.27
$\xi_1 = 0.03$ $u = 0.004$ $\eta_1 = 0.002$ $v = 0.01$ $\xi_2 = 0.006$ $\eta_2 = 0.005$ $N_0 = 1000$ $N = 3000$ $\sigma = 100$ $c_2 = 0.3$	280	1490	0.188	0.188
$\xi_1 = 0.6$ $u = 0.004$ $\eta_1 = 0.005$ $v = 0.01$ $\xi_2 = 0.1$ $\eta_2 = 0.005$ $\sigma = 100$ $c_2 = 0.3$	24.6	144.6	0.17	0.17

Table 3.3 Simulation results of sample means for $N(t) = N_0 + (N_1 + \varepsilon_t)(1 - e^{-c_2 t})$

Now we examine the model when the parameters are stochastic. When the parameters of the diffusion equations are stochastic, that is:

$$\begin{cases} \dot{x}_1(t) = -\hat{\xi}_1 x_1(t) + \hat{\eta}_1 (N(t) - x_1(t) - x_2(t)) + u x_2(t) \\ \dot{x}_2(t) = -\hat{\xi}_2 x_2(t) + \hat{\eta}_2 (N(t) - x_1(t) - x_2(t)) + v x_1(t) \end{cases}$$

, where

$$\begin{aligned} \hat{\xi}_1 &\sim N(\xi_1, \sigma_1^2), \hat{\xi}_2 \sim N(\xi_2, \sigma_1^2) \\ \hat{\eta}_1 &\sim N(\eta_1, \sigma_1^2), \hat{\eta}_2 \sim N(\eta_2, \sigma_1^2) \\ \hat{u} &\sim N(u, \sigma_1^2), \hat{v} \sim N(v, \sigma_1^2) \end{aligned}$$

Here we assume σ_1 is smaller enough so that $\hat{u} \geq \hat{\eta}_1$, $\hat{v} \geq \hat{\eta}_2$ almost always holds

Example 5: In this example, we assume

$$\begin{aligned} \xi_1 &= 0.03 \quad u = 0.004 \quad \eta_1 = 0.002 \\ v &= 0.01 \quad \xi_2 = 0.006 \quad \eta_2 = 0.005 \\ N_0 &= 1000 \quad N = 3000 \quad c_2 = 1 \\ \frac{\bar{x}_1}{\bar{x}_2} &= \frac{\xi_2 \eta_1 + u \eta_2}{v \eta_1 + \xi_1 \eta_2} = 0.1882 \end{aligned}$$

We list the simulation results in table 3.5, 3.6 when $\sigma = 0.005$, $\sigma = 0.01$ respectively. The results show that the demands on both channels are stable on equilibrium.

Simulations indicate that the proposition and theorem still hold even if the market size is not deterministic and the diffusion parameters are stochastic. The simulation study suggests the results given by Yao & Liu under the condition of deterministic demand can be applied to the situation where demand is uncertain. Thus this simulation study is the valuable extension of their research results.

Initial condition	\bar{x}_1	\bar{x}_2	\bar{x}_1 / \bar{x}_2	95% CI
$x_1(0)=0, x_2(0)=50$	290	1506	0.1926	(0.18495, 0.20047)
$x_1(0)=0, x_2(0)=300$	284	1480	0.1918	(0.18385, 0.20067)
$x_1(0)=0, x_2(0)=800$	270	1487	0.1815	(0.17118, 0.19234)

Table 3.4 Redistribution Solutions when $\sigma = 0.005$

Initial condition	\bar{x}_1	\bar{x}_2	\bar{x}_1 / \bar{x}_2	95% CI
$x_1(0)=0, x_2(0)=50$	282	1450	0.1944	(0.17553, 0.21223)
$x_1(0)=0, x_2(0)=300$	242	1483	0.1628	(0.1434, 0.1850)
$x_1(0)=0, x_2(0)=800$	301	1520	0.1979	(0.18096, 0.21627)

Table 3.5 Redistribution Solutions when $\sigma = 0.01$

SUMMARY

In this paper, we take the simulation approach to study the effect of market characteristics and parameters variability on the channel redistribution extended from the previous research. Our findings indicate no matter the market size follows exogenous random disturbance, stochastic aggregate fluctuation, or the diffusion parameters vary, the ratio of demand on direct channel to the demand on indirect channel is same when the redistribution is stable. So the results of demand redistribution when market size is deterministic are pretty robust, it really provides managerial implications for the channel of distribution design issue.

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