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VISUAL MODELING OF BUSINESS SIMULATIONS

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ABSTRACT

This paper presents a visual modeling technique which will aid designers of business simulations. Three demand examples are presented using the visualization software, Mathematica.

INTRODUCTION

Over the past twenty years ABSEL model builders have worked diligently to improve the algorithms that drive business simulations. ABSEL researchers such as Gold, Pray Goosen, Teach, Decker, LaBarre, Thavikulawat, and Carvallo have published numerous papers on different aspects of improving the realism and the reliability of business games. Still, many designers, even after developing equations, struggle with: (i) how to select starting values, (ii) how to gauge sensitivity of the parameters used in the model, and (iii) how to ensure the system is robust. This paper is intended to illustrate a modeling and visualization tool known as Mathematica that can be used by designers of business simulations to gain a better understanding of their models.

The paper begins with a brief summary of the ABSEL and Simulation and Gaming literature on algorithm development and then gives a general description of Mathematica describing, what it is and how it can be used. Finally, it offers a "visual modeling technique." Three commonly used demand models are analyzed. Visual exploratory analysis is performed on (i) the Cobb Douglas power function, (ii) the Gold and Pray Demand System (1984), and (iii) the product attribute model of Gold and Pray (1997). Mathematica is utilized to identify both stability and lack of stability of a system of equations. The paper concludes with some suggestions on the use of the software package and caveats associated with the methodological approach suggested by the authors.

BUSINESS SIMULATION ALGORITHMS

A review of the modeling literature illustrates that there has been considerable work in algorithm enhancement of business games. For example, in the operations arena, Thavikulaw (1993) proposed a linear set of equations to model production processes. Gold and Pray (1989), Gold (1990), and Gold (1992) have developed models for cost and production functions embodied in business games. Pray and Methe (1991) put forth a model for new product development with generalized demand and production functions.

Quality modeling became popular in the 1990s with the work of Thavikulwat (1992), Mergen and Pray (1992) and Teach (1992). All of these authors demonstrated methods and algorithms for modeling quality that could be added to existing or to new simulations.

In the area of marketing, many articles have been written about how to model demand and other non-price determinants of demand. Pray and Gold (1982) with their classic article "Inside the Black Box" investigated the demand robustness of a number of commonly used business games. Articles soon followed by Teach (1984), Gold and Pray (1984), Goosen (1986) and Decker, LabBarre and Adler (1987). These authors' work on algorithms moved the modeling of demand to a higher level. Further extensions by Golden (1987), Lambert and Lambert (1988) and Thavikulwat (1988,89) tested the reliability of various models and raised new issues about how demand should be modeled. Market segmentation was addressed formally by Teach (1990), Carvalho (1991, 95) and Gold and Pray (1997,98).

As can be seen by this brief review, the leading business games designers have shared their design contributions with the field. But all of these algorithms described in the literature are just mathematical models and thus have certain limitations and shortcomings. Some algorithms are highly sensitive to the starting parameters selected. Others require the decision variables to be constrained in a narrow range for the simulation to behave in a manner that is consistent with theory. Some models have discontinuities, which can also yield unreasonable results. To identify the shortcomings in a model, one alternative is to look at lots of numerical results while manipulating a few variables. A better alternative, in many cases, would be to look at a visualization of the model and to thus explore its behavior across a variety of situations.

VISUALIZATION

Visualization is a process in which images are created to gain new insight into abstract data or complex functions. Much of the visualization work to date has been for scientific applications like weather forecasting, fluid dynamics, or Magnetic Resonance Imaging (MRI). However, recent applications like data mining, process streamlining and network analysis have driven new demand for visualization in the Business environment.

For simulation designers, visualization offers a range of tools and techniques that will allow them to explore their complex, many-dimensional models. The methodology to be presented here offers a relatively easy “visual method” of testing and verifying the overall effectiveness of the algorithm, and provides insights into where difficulties may arise with actual usage. The drudgery of hours of mathematical sensitivity analysis can be avoided with the visual approach promoted by Mathematica.

MATHEMATICA

Mathematica™, by Wolfram Research, is a software tool that allows the creation, solution, visualization and distribution of complex mathematical models. Almost every conceivable mathematical operation, analysis or function can easily be conducted with this software. In general, Mathematica finds an analytical solution for a huge variety of equations. When this is not possible, several different approximation techniques are available to provide numerical solutions for a user-specified error tolerance. The built-in mathematical functions often enable solutions for problems that would be very difficult or impossible for most users without this tool.

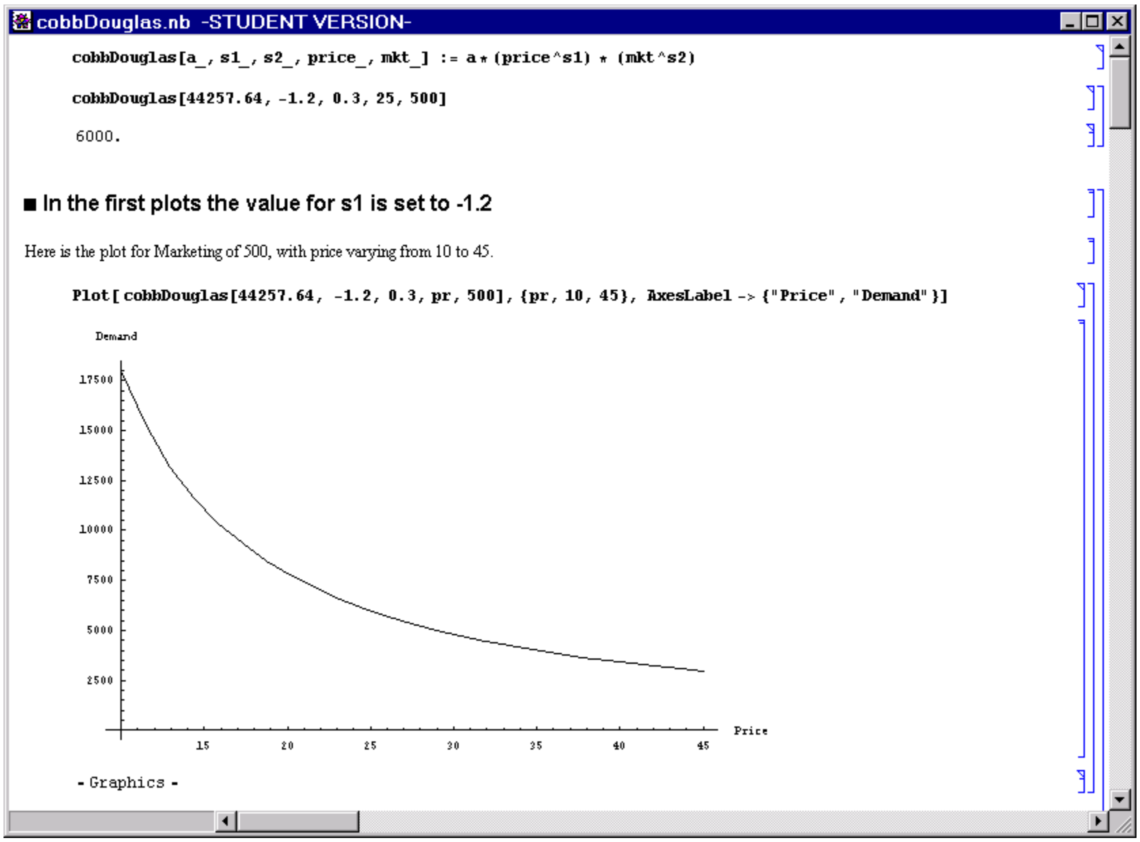
WORKING WITH MATHEMATICA

The Mathematica interface is an electronic notebook, where one can include ideas, partial results, and graphics. Users develop mathematical models by evaluating individual lines of Mathematica code, thus creating partial results that can be combined over and over again to develop more complex models. Visualization functions are available at every step of the development process to help a model-builder verify the behavior of the model. Because of the variety of tools and functions available, a developer will frequently discover unanticipated behaviors that can enhance his understanding or help him avoid future problems with a model.

Once most of the development has been completed, the notebook interface can be used to explore the model both numerically and visually. This exploratory mode of interaction with Mathematica can take the form of a set of “what-if” scenarios that allow the user to explore the full complexity of the work. Charts, graphics and animations can be created automatically to contribute to the user’s understanding. Sharing such a model online is simple since the freely available MathReader software allows anyone to explore and interact with a notebook.

Figure 1 show portions of Mathematica in action. A Cobb Douglas demand function with the elasticities and scale parameters are set so that the demand would be 6000 units at the starting values. Varying only the price then generates a two-dimensional demand plot.

FIGURE 1
WORKING WITH MATHEMATICA



LEARNING MATHEMATICA

Like any new piece of software, a new user to Mathematica will have to spend some time learning both the notebook interface and the language used to write equations and functions. While the notebook interface is quite easy, the language of available commands is vast and can be intimidating. Fortunately, one can accomplish most analyses by exploring just the small subset of the language that is applicable to a specific problem. The documentation for Mathematica is very well written and illustrated, and additional reference guides make developing significant models possible even for beginners.

VISUAL MODELING TECHNIQUE

What follows is a demonstration of the visual modeling and exploratory techniques using three different demand models.

Cobb Douglas Market Demand Function - A Stable Function with Constant Elasticity

The first model selected is the Cobb Douglas function. This function was first deployed as a way to describe production functions in microeconomics, but can be easily modified to fit the demand side. To demonstrate the visual modeling aspects, we will simulate a simple demand function where price (P) and marketing

(M) are the independent variables, and demand (Q) is the response or dependent variable.

The functional form is as follows:

$$Q=aP^{-ep}M^{em} \quad (1)$$

where: the elasticities for price and marketing are ep and em respectively. "a" is scaling coefficient.

To verify the coding of the model we set the price elasticity (ep) at –1.2, the marketing elasticity (em) at .3, and the scaling coefficient "a" at 44257. As in Figure 1, using these values, the demand will start about 6000 units for a price of \$25 and marketing expenditure at \$500. We then varied price from \$3 to \$45 and marketing from \$200 to \$3000. The results are demonstrated below using Mathematica.

FIGURE 2
THE COBB DOUGLAS DEMAND CURVE

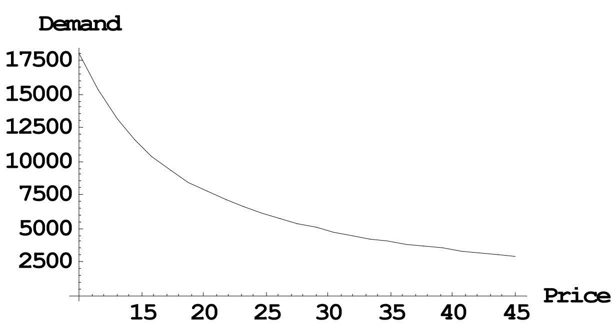


Figure 2 shows a two dimensional plot - a classic Marshallian demand curve. In this example price was varied from \$3 to \$45 while marketing was held constant at \$500. The plot shows that demand is maximized at 17,500 and that the quantity demanded appears to be asymptotic to the x-axis.

In the next illustration, Figure 3, the diminishing returns to marketing are clearly seen as we vary marketing from \$200 to \$3000 while holding price constant at \$20. It is interesting to note that demand reaches zero and that for this model it is possible to generate "negative demand " for very small levels of marketing.

FIGURE 3
DIMINISHING RETURNS TO MARKETING

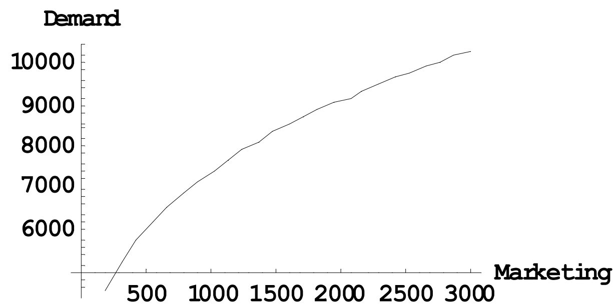
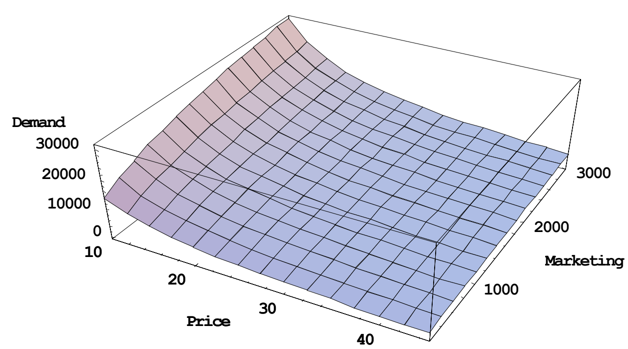


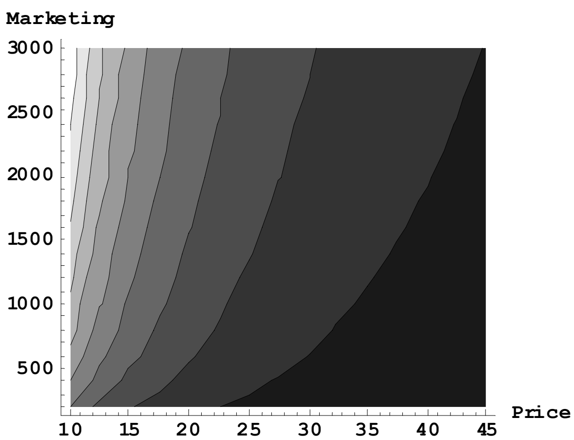
FIGURE 4
THE DEMAND SURFACE



To look at systems of more than two variables, Mathematica allows three-dimensional plots. In the Figure 4, we vary both price and marketing. Figure 4 illustrates the non-linearity of the demand function and the relative stability of behavior over the range. What is interesting to note is that at high prices, over \$40, even large dollar expenditures in marketing will **not** increase demand very much. This may be construed as a shortcoming of the model, possibly indicating that high-priced niche strategies may not be successful with this demand model.

A contour map, Figure 5, provides another way to look at the effects of two variables. In this graphic, the lighter areas represent higher levels of demand. Again, notice the lack of sensitivity when price is high. Although difficult to see when looking at tables of numbers, this shortcoming of the demand function becomes apparent with the visual approach. In fact, the diminished impact of marketing expenditure at high prices was not discovered by the authors until they viewed these plots.

FIGURE 5
THE CONTOUR MAP



Gold and Pray - A Variable Elasticity Market Demand Function

Gold and Pray (1984) developed a variable elasticity demand model to overcome the shortcomings of using the Cobb Douglas system. Their system involves 10 equations and is described in detail with examples in Gold and Pray [1984]. The algorithm to be simulated is as follows:

$$Q_t = g_1 P_t^{-(g2+g3Pt)} M_t^{+(g4-g5Mt)} \qquad (2)$$

where: Q_t = market demand at *time t*, P_{jt} = average price at *time t*, M_{jt} = average marketing expenditure at *time t*, and g_k = market demand parameters k where k = 1 through 5.

To solve for the parameters of the market demand equation the administrator must specify the desired exogenous elasticities of each independent demand variable at two different levels (i.e. P,M). The elasticity formulas are as follows:

$$Ep_t = g2+g3P_t\,(1 + \ln P_t) \qquad (3)$$
$$Em_t = g4+g5M_t(1 + \ln M_t) \qquad (4)$$

where: Ep_t = price elasticity at *time t,* Em_t = marketing expenditure elasticity at *time t*.

Selecting two levels for each elasticity (E_p, and E_m) and the corresponding demand variable (P and M) over a reasonable range gives two equations with two unknowns and allows simultaneous solution of the system parameters g_k (for k=2,5). The selection of g1 determines the initial market size. To demonstrate, the following values were set:

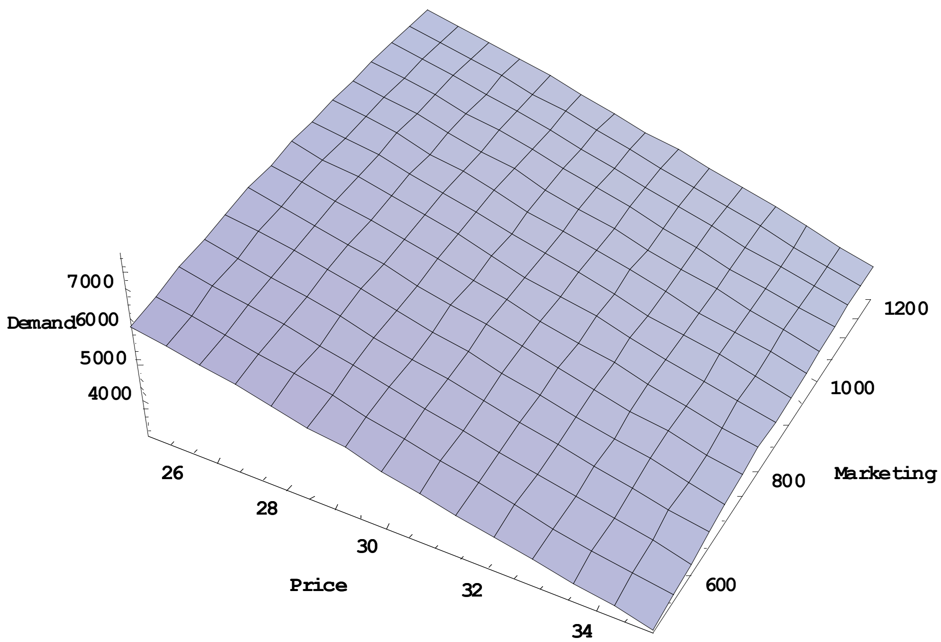
FIGURE 6
THE PARAMETERS

	Starting value	Final value
Price	$ 25	$ 35
E_p	.95	3.00
Marketing	$500	$1500
E_m	.4	.15

With these parameters, the price elasticity of demand at the market level will increase from an inelastic .95 to a highly elastic 3.00 when the price increases from $25 to $35. Likewise, the marketing elasticity declines as more money is allocated to marketing.

With these values and a scaling coefficient, market demand is about 6000 units when price is $25 and marketing $500. Using Mathematica and varying price over the relevant range from $25 to $35 and marketing from $500 to $1500 yields some very interesting results.

FIGURE 7
GOLD AND PRAY DEMAND SURFACE



The 3-D plot of Figure 7 demonstrates that the gross behavior of the Gold and Pray model is consistent with the theory reflected in the Cobb Douglas function. It is interesting to note that this variable elasticity behaves similarly to that in the Cobb Douglas model in that at the higher prices, demand is **not** very responsive to increases in marketing.

The Gold and Pray model can be further verified by using Mathematica to calculate the arc elasticities for price and marketing. Indeed, this line graph shown in Figure 8 shows the price elasticities are consistent with expectations. However, the model and theory are in agreement only if the price and marketing values are constrained to be within the relevant ranges of Figure 6.

FIGURE 8
PRICE ELASTICITIES

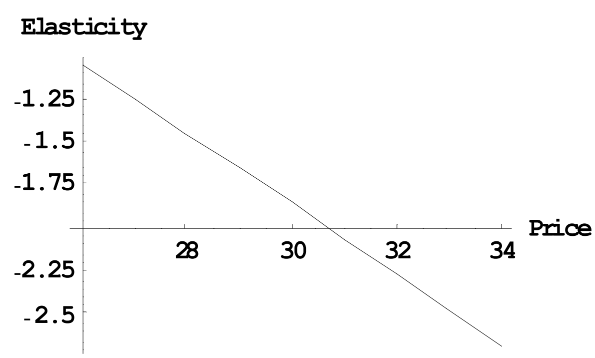
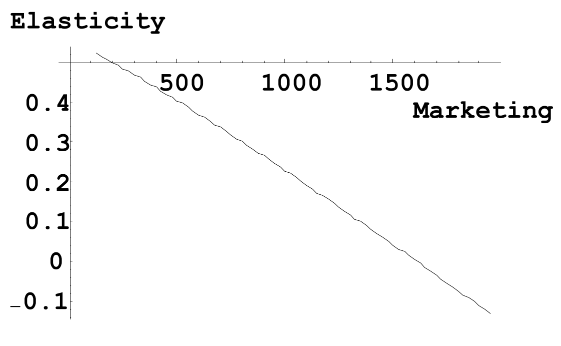


Figure 9 shows that the marketing elasticity is consistent with theory over the range \$500 to \$1500. However, above \$1500 negative returns occur to marketing.

FIGURE 9
MARKETING ELASTICITY

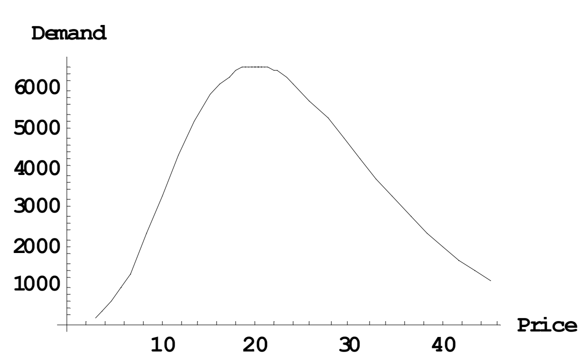


Instability of Function - Price Issues

Mathematica makes it readily apparent that the demand system is robust over the constrained range of decision inputs. Increasing the price and marketing out of the range reveals some fascinating behavior about the model.

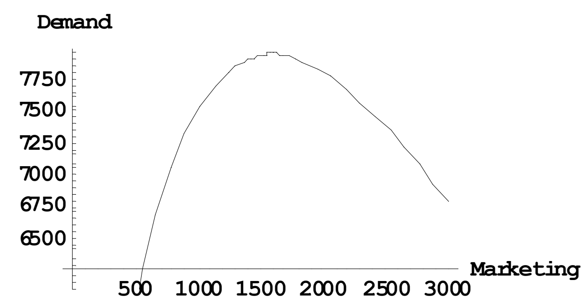
Notice in Figure 10 that at prices from \$4 to \$20 the function behaves in a manner that is **inconsistent** with demand theory. The model behaves consistently with theory for all prices above \$20. What is interesting however, is that at lower prices say from \$5 to \$18 dollars, price increases cause demand to increase! We don't

INSTABILITY IN THE FUNCTION
FIGURE 10



think the designers expected the model to behave

FIGURE 11
INSTABILITY OF THE FUNCTION-MARKETING ISSUES



as an economic Giffen good.

Figure 11 depicts quantity demanded with advertising varying from \$200 to \$3000 and price fixed at \$25. The marketing response appears to be consistent with expectations over the relevant range. But negative returns to marketing occur outside the upper limit of \$1500. The negative returns may be helpful in some simulation situations. But the theoretical realism of negative returns can be challenged.

The Gold and Pray Attribute Model

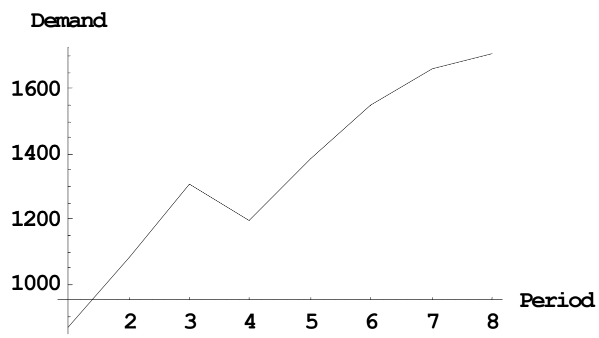
The final system is described via a set of simple examples. To determine the market- and firm-level demand for each segment, Equation 5 is used for each segment. Price is set to vary between \$25 and \$35 and advertising/marketing expenditures, between \$500 and \$1200. The elasticities of the gravity flow distances (d_{ijt}) ,based on the scales, are controlled over a range from 0 to 5.

$$Q_{jt} = g_1 P_{jt}^{-(g2+g3Pjt)} M_{jt}^{+(g4-g5Mjt)} D_{jt}^{-(g6+g7Djt)} \quad (5)$$

where: Q_{jt} = market demand for the segment j at *time t*, P_{jt} = harmonic average price of all products in segment j at *time t*, M_{jt} = average marketing expenditure for all products in segment j at *time t*, D_{jt} = average distance of all products for the segment j at *time t*, g_k = market demand parameters k.

Gold and Pray (1999) demonstrated how distance could be employed in the model to simulate attributes desired by the customer. In their example they illustrated that Firm 3 would gain significant market share by introducing a new product based on attributes desired by the customer. In Figures 12 and 13 we replicated Gold and Pray's example of the new product introduction but added a new element. We

FIGURE 12
FIRM 3'S DEMAND
PRICE INCREASED IN PERIOD 4



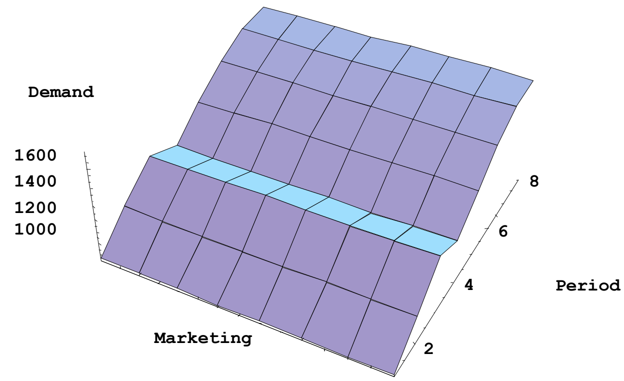
assumed the firm would increase price from $25, to say $28, because of the added features. Mathematica was then utilized to visually present firm's demand with ceterus paribus.

Notice the rapid decline in demand at period 4, but the demand quickly responds, demonstrating the model allows for firms to increase their price substantially with the new features. Figure 13 shows the result in a three-dimensional perspective.

Summary and Conclusions

The purpose of this paper was to present a visual modeling technique that we recommend to designers of business simulations. Utilizing visual representation will aid designers in creating their models by showing the parameter

FIGURE 13
THREE-DIMENSIONAL VIEW



configurations where the model might behave unstably. Furthermore, the interactive nature of the Mathematica tool allows designers to try many different configurations quickly to help identify those that will be used in the final algorithm.

In the first of three examples presented, the visual modeling technique helped identify the shortcomings of the Cobb Douglas as a model for demand modeling. Specifically, it was noted that the impact of marketing expenditure is greatly diminished at high prices. In our second example, the Gold and Pray demand system was shown to resolve some of the shortcomings (by allowing the elasticities to vary), but was highly unstable outside the preset parameters. The final illustration took the Gold and Pray (1999) attribute model and illustrated what happened to firm-level demand when the firm introduced a new product and simultaneously increased the product price. It suggests that the third model may alleviate some of the shortcomings of the previous two models.

References available upon