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MODELING SHORT-RUN COST AND PRODUCTION FUNCTIONS USING SHEPPHERD'S LEMMA IN COMPUTERIZED BUSINESS SIMULATIONS

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ABSTRACT

The paper develops an algorithm to model short-run cost and production functions using Sheppard's lemma in computerized business simulations. The algorithm is derived utilizing duality theory to maintain consistency between the production technology and cost relationships of the firm. The short-run cost function is shown to depend on: variable factor prices, the production rate, and the level of the fixed factors. Sheppard's Lemma is applied to derive the cost minimizing input demand levels based on the characteristics of the short-run cost function. A recommended system of equations is presented and discussed to simulate the theoretical model of the firm. A numerical example is given to illustrate how the parameters of the equation set can be estimated and how the functions behave. The system is shown to be flexible and may be applied to model a wide array of cost structures.

THE PROBLEM AND PURPOSE

In a seminal paper, Kenneth Goosen (1981) encouraged simulation designers to be more open about the way in which they have modeled their simulations. He noted that prior to 1981 only sixteen professional papers dealt with design concerns in creating simulations. Identifying the algorithms embodied in simulations would be helpful not only to individuals interested in developing simulations but also to users of simulations.

Users of simulations are sometimes puzzled about the results of a team's play. Numerous questions are raised about simulation performance, such as: "Why did my profits fall? What caused my costs of goods sold to rise so quickly? or Why was there such a substantial decline in my stock market value?" Although some design issues have been addressed in recent years they have pertained primarily to the demand side (see Decker, LaBarre, & Adler(1987); Frazer(1983); Gold & Pray(1983, 84), Golden(1987), Goosen(1986); Lambert & Lambert(1988); Teach(1989, 1990); and Thavikulwat(1988)). A paper by Thavikulwat(1989) was one of the first attempts at carefully modeling the supply side of the firm; and he stated:

"The problem of modeling supply... has tended to be neglected. Yet, the supply side of modeling presents issues that are at least as involved as those of the demand side." (p.37)

In this paper the design and relationship of production and cost structures in computerized business simulations will be addressed. According to a study by Whitney, et. al. (1990) the management of technology is becoming a focus of attention in many business schools. This raises an interesting question as to how technology is modeled in business and management simulations. A review of a number of contemporary business simulations by Gold and Pray (1989) identified a problem in the design of the production technologies in a wide array of business simulations. Almost all the simulations reviewed displayed a linear relationship between production and costs in both the short-run and long-run, implying constant returns to the variable inputs and no economies-of-scale in the cost structure. This is in sharp contrast to numerous economic studies which have shown that economies of scale are widespread in industry (see Walters (1963)).

Gold and Pray (1989) also noted that there were inconsistencies between the modeling of production and the implied cost structure of the simulated firms. Some designers modeled economies of scale by changing input prices while keeping productivity constant. While this approach may seem adequate it allows productivity to be constant and, simultaneously, average costs to decline. Although this result is possible, it is not the general case. Duality theory argues that economies of scale are derived, more generally, from increasing returns in the production process given fixed factor prices.

In an attempt to address this concern Gold(1990)

developed a system to model the cost structure of the firm in a manner consistent with duality theory. However, the cost structure developed by Gold was a long-run analysis and assumed all factors of production were variable. Since all firms operate with fixed constraints in the short-run it is important to assess the impact of plant size and capacity constraints on the costs of the firm. The purpose of this paper is to address this concern and incorporate the short-run with fixed factors of production in the modeling of cost and functions.

METHODOLOGY

The approach taken in this paper allows the designer to specify the cost relationships of the firm first and then derive, jointly, the levels of input usage and the production function implied by the cost structure. The advantages of this approach are twofold. First, it guarantees that the behavior of the production function will be consistent with the cost structure of the firm. Second, it utilizes cost information rather than production data to model the firm. Cost information is more accessible in published sources than production data. Since the methodology in this paper uses cost information to develop the cost function, first, and then derives the implied production technology, it is easier to simulate and does not require production data.

More specifically the methodology involves:

- (1) Deriving a generalized short-run cost function based on the theoretical properties of duality theory and Sheppard's Lemma. The short-run is carefully distinguished from the long-run cost characteristics of the firm.
- (2) Developing a recommended system of equations for modeling short-run cost and production functions. The recommended system of equations maintains the dual relationship between the production and cost characteristics of the firm.
- (3) Specifying a procedure to estimate the parameters of the equation system based on the apriori specifications of the designer. A numerical example is given to illustrate the procedure and show the characteristics of the functional form.
- (4) Simulating the equation set and deriving the implied production technology given the estimated parameters of the system. The numerical example is used to illustrate how the system behaves.

DERIVING THE GENERALIZED SHORT-RUN COST FUNCTION

The cost function depends on the production technology of the firm. Assuming one fixed input, capital equipment, and two variable inputs, labor and materials, we can express the general production function as:

$$Q = f(K, L, M) \quad (1)$$

where

Q = quantity produced	(units)
K = capital equipment	(units)
L = labor	(hours)
M = material	(pounds)

subject to the short-run constraint that capital is fixed:

$$K = K = \text{fixed} \quad (2)$$

In the short-run the total costs of the firm may be divided into fixed and variable costs.

$$TC = TFC + TVC \quad (3)$$

$$TFC = (P_K)K \quad (4)$$

$$TVC = (P_L)L + (P_M)M \quad (5)$$

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where: TC total costs (\$/period)
 TFC = total fixed costs (\$/period)
 TVC = total variable costs (\$/period)
 Pk = price of capital (\$/unit)
 P1 = price of labor (\$/hour)
 p3 = price of mat'l (\$/lb.)

Dividing both sides of the cost equations (3, 4, 5) by the quantity produced Q, we get:

$$ATC = AFC + AVC \quad (6)$$

$$AFC = Pk/APk \quad (7)$$

$$AVC = P1/AP1 + Pm/AP \quad (8)$$

where: ATC = average total costs
 AFC = average fixed costs
 AVC = average variable costs
 APk = average product of capital = Q/K
 AP1 = product of labor = Q/L
 AP = average product of mat'l = Q/M

These equations (6-8) illustrate the dual relationship between production and cost in the short-run. Average total cost (ATC) is the sum of average fixed cost (AFC) plus average variable costs (AVC). Average fixed costs and average variable costs are inversely related to the average products of the respective inputs. As the average product of capital (APk) rises, the AFC will decline. As the average product of labor (AP1) or material (AP) increase the AVC will decrease.

The short-run marginal cost equation may be derived by taking the derivative of TVC with respect to Q since the marginal product of the fixed input, K, is zero:

$$MC = P1/MP1 + P/MPm \quad (9)$$

where: MC = marginal cost of production
 MP1 = marginal product of labor = dQ/dL
 MPm = marginal product of material = dQ/dM

Equation 9 shows short run marginal costs (MC) are inversely related to the marginal products of the variable inputs. The marginal product of the fixed input is zero in the short-run and is not included. As the marginal product of labor or materials increase, the MC will decline.

Applying the approach presented by Sheppard (1970) it may be shown that costs can be expressed as a function of the level of input prices and production. The first step is to formulate the lagrangian equation for minimizing total variable costs, equation 5, subject to a given level output, the production function, equation 1, and the fixed input constraint, equation 2:

$$Z = (P1)L + (Pm)M + q(Q - f(L,M,K)) + k(K - K^*) \quad (10)$$

where: q = lagrangian multiplier of constraint Q
 k = lagrangian multiplier of constraint K

The cost minimizing input usage may then be obtained by setting the partial derivatives of the lagrangian equation (Z) with respect to the inputs equal to zero, giving us the following first order conditions:

$$dz/dL = P1 - q (dQ/dL) = 0 \quad (11)$$

$$dz/dM = Pm - q (dQ/dM) = 0 \quad (12)$$

$$dz/dq = Q - f(L,M,K) = 0 \quad (13)$$

$$dz/dk = K - K^* = 0 \quad (14)$$

Solving the equation set simultaneously, and substituting into equation 1, the generalized cost function may be written as:

$$TVC = f(P1, Pm, Q, K) \quad (15)$$

Equation 15 shows that total variable costs may be expressed as a function of input prices, production, and the level of capital, without directly specifying the level of input use.

The level of input use may be derived by applying Sheppard's Lemma. Sheppard (1970) proved that the demand for the variable inputs may be obtained by differentiating the cost function with respect to the variable input prices. Given the generalized cost function, equation 15, and applying Sheppard's Lemma we get:

$$L = d(TVC)/dP1 \quad (16)$$

$$M = d(TVC)/dPm \quad (17)$$

Equation 16 specifies that the quantity of labor used

by the firm may be determined through the cost function by taking the derivative of total variable costs with respect to the price of labor. Similarly, equation 17 specifies the quantity of material used by the firm is the derivative of total variable costs with respect to the price of materials.

Sheppard's Lemma is a powerful theoretical tool for the design of cost and production functions. Once the cost function is specified, the demand for inputs (labor and materials) may be ascertained in a manner consistent with duality theory. In this case, increases in average variable costs or marginal costs would imply decreases in average products or marginal products of the variable inputs (as described by equations 8 & 9).

The output elasticity or returns to the variable input (E) is measured by the ratio of short-run AVC to MC, such that:

$$E = AVC/MC \quad (18)$$

If $E > 1$ then increasing returns to the variable input exists. Increasing returns implies AVC exceeds MC. If $E < 1$ then decreasing returns exist, indicating MC exceeds AVC. If $E = 1$ then there are constant output elasticities, $AVC = MC$, and AVC is minimized. Generally, it is expected that at low levels of output the firm would be able to achieve increasing returns, and after some point of diminishing returns would only be able to obtain decreasing returns to the variable input.

A SHORT-RUN COST SYSTEM

For clarity of exposition, two variable inputs and one fixed input will be used to illustrate the function, but the model may be easily generalized to any number of arguments. The short-run cost function is multiplicative in nature and is flexible enough to model increasing and decreasing returns to the variable input, given a fixed level of capital:

$$TVC = a1(P1)^{a2} (Pm)^{a3} Q^{(a4 + a5Q - a6K)} K^{a7} \quad (19)$$

$$TC = TVC + (Pk)K \quad (20)$$

where: TVC = total variable cost
 TC = total costs (variable + fixed costs)
 Ai = parameters; i = 1 to 7
 P1 = price of labor input
 Pm = price of mat'l input
 Q = quantity produced
 K = capital equipment

An important characteristic of equation 19 is that total variable costs, TVC, depend on the level of the fixed factor, K. Although this adds to the complexity of the function, it makes the function more realistic and follows from the assumption of non-separability between variable and fixed factors of production. As explained by Bernt and Christensen (1973) non-separability exists within a function when the marginal rate of substitution between any two variable factors (L and M) is dependent on the fixed factor(s), K. This assumption is required to embody economies or diseconomies of scale in the cost structure. Economies of scale implies that AVC will decline in the long-run with increase in capital, K. A study by Walters (1963) concluded that economies of scale is characteristic of virtually all manufacturing operations.

The multiplicative functional form of the cost equation makes the parameters relatively easy to interpret, and is a stable and flexible function for simulation purposes. The parameter a1 is simply a scaling factor to obtain the desired level of cost. Parameters a2 and C are the input price elasticities and show the proportion of input costs to total variable costs. The exponent term $(a4 + a5Q - a6K)$ associated with Q, allows for variable cost elasticities with respect to both output, Q, and capital, K. Variable cost elasticity is necessary to model increasing and decreasing returns to the variable inputs. The cost elasticity increases as the level of output increases or the level of capital decreases. Finally, parameter a7 impacts the capital elasticity, which determines, in part, the sensitivity of TVC to changes in the level of the fixed factor.

The output elasticity (E) may be derived from equation 18 since $E = AVC/MC = (TVC/Q)/(TVC/dQ)$.

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Economies of Scale

$$E = 1/(a_4 + a_5 Q^{1.0} + \ln Q) - a_6 K \quad (21)$$

where: E = economies of scale
 $\ln Q$ = natural log of Q

The resulting input demand equations derived from the cost function, using Sheppard's Lemma for the variable factors are:

Input Demand Equations

$$L = a_2 (TVC)/P_l \quad (22)$$

$$M = a_3 (TVC)/P_m \quad (23)$$

$$K = K^* \quad (24)$$

Capital, K, is assumed to be fixed in the short-run. A well behaved cost function also requires the following restriction:

Homogeneity Restriction

$$a_2 + a_3 = 1.0 \quad (25)$$

This restriction guarantees that the total variable cost function is homogenous of degree one. This simply means that if all variable input prices increase by some proportion, say 10%, then total variable costs will increase 10%, given a constant production level. This relationship holds by definition, refer to equation 5.

NUMERICAL EXAMPLE: ESTIMATING THE PARAMETERS OF THE SYSTEM

One of the advantages of the multiplicative functional form, is the ease in which parameters of the system can be estimated given the design characteristics of the simulated cost function. To illustrate how the parameters may be estimated, a numerical example will be given.

Suppose a simulation designer wants to model a cost function that possesses increasing returns to the variable inputs of 1.5 at an initial output level of 1000 units, and constant returns of 1.0 (the point of minimum AVC) at 1500 units of output, given a fixed level of capital at 2000 units. If the level of capital is increased to 2200 units, the point of constant returns or minimum AVC would be extended to 1600 units of output. Of course, any scenario consistent with standard cost behavior could be designed. This example implies the following data specifications:

Output Elasticity	Output(O)	Capital (K)
E = 1.5	1000 units	2000 units
E = 1.0	1500 units	2000 units
E = 1.0	1600 units	2200 units

According to the homogeneity restriction, the sum of the proportion of variable costs that are attributed to labor (a₂) and the proportion of variable costs attributed to materials (a₃) must sum to 1.0. Further suppose the capital elasticity parameter, a₇, is specified to be 0.5. To summarize:

Variable Inputs	Parameter	Fixed Inputs	Parameter
L	a ₂ = 0.80	K	a ₇ = 0.50
M	a ₃ = 0.20		
Sum	1.00		

Finally, the designer needs to specify the total variable costs corresponding to the initial output level of 1000 units; and the input prices:

TVC	= \$28,490
P _l	= \$25
P _m	= \$10
P _k	= \$1

Given the above data, the first step is to solve for the parameters a₄, a₅, and a₆ by using equation 21 and substituting in the values for E and Q above.

$$1.5 = 1.0 / (a_4 + a_5(1000) (1.0 \cdot \ln 1000) - a_6(2000));$$

$$1.0 = 1.0 / (a_4 + a_5(1500) (1.0 \cdot \ln 1500) - a_6(2000));$$

$$1.0 = 1.0 / (a_4 + a_5(1600) (1.0 \cdot \ln 1600) - a_6(2200));$$

Simplifying each equation respectively:

$$0.667 = a_4 + 7907.76 a_5 - 2000 a_6;$$

$$1.000 = a_4 + 12469.83 a_5 - 2000 a_6;$$

$$1.000 = a_4 + 13404.41 a_5 - 2200 a_6$$

Solving the three equations simultaneously:

$$a_4 = 0.7174$$

$$a_5 = 0.000073066$$

$$a_6 = 0.00034143$$

The second step is to solve for the parameter a₁. Simply substitute all known parameters and variables into equation 19 and solve. At this point all parameters and variables are known except for a₁ (recall a₇ was specified by the designer):

$$28490 = a_1(25)^{0.8} (10)^{0.2} (.71 + .000073 * 1000 - .000341 * 2000) \cdot .5 * 1000 \quad 2000^5$$

Solving the above equation for a₁ we get: a₁ = 10.0.

Note that a₁ is a scaling factor and does not affect the shape or properties of the short-run cost function. The final total variable cost equation becomes:

$$TVC = 10(P_l)^{0.8} (P_m)^{0.2} Q(.7174 + .000073 * Q - .0003414 * K) \cdot .5 K$$

Once the parameters of the cost function are known, the input levels for labor and materials may be derived by substituting into equations 22 and 23 the values for TVC given any level of Q, P_l, P_m, and K.

$$L = 0.80 (TVC)/25$$

$$M = 0.20 (TVC)/10$$

The input levels derived in this manner are consistent with the "dual" relationship between cost and production. Since TVC depends on the level of K, Q, P_l, and P_m, the input demands, L and M, are also a function of these factors.

SIMULATING THE SYSTEM

The short-run cost and "dual" production system will be simulated using the estimated parameters in the numerical example above to illustrate its characteristics: and to demonstrate the relationship between variable costs, fixed costs, and the production function.

The simulation first assumes capital, K, is fixed at 2000 units and then increases capital to 2200 units. The production rate is varied between 1200 units and 1800 units, in increments of 100 units. Factor prices are fixed at \$25, \$10, and \$1 for labor, materials, and capital respectively. Tables 1 and 2 summarize the cost characteristics of the system..

TABLE 1
Impact of changing Output Levels on Costs given Capital fixed at 2000 units

Output	TFC	TVC	TC	AFC	AVC	ATC	E
1200	2000	32547	34547	1.67	27.12	28.79	1.25
1300	2000	34786	36786	1.54	26.76	28.30	1.16
1400	2000	37180	39180	1.43	26.56	27.99	1.07
1500	2000	39741	41741	1.33	26.49	27.83	1.00
1600	2000	42483	44483	1.25	26.55	27.80	0.94
1700	2000	45419	47419	1.18	26.72	27.89	0.88
1800	2000	48564	50564	1.11	26.98	28.09	0.83

$$* TFC = (P_k)K = (1) (2000) = 2000$$

TABLE 2
Impact of changing Output Levels on Costs given Capital fixed at 2200 units

Output	TFC	TVC	TC	AFC	AVC	ATC	E
1200	2200	21035	23235	1.83	17.53	19.36	1.37
1300	2200	22359	24559	1.69	17.20	18.89	1.26
1400	2200	23777	25977	1.57	16.98	18.56	1.16
1500	2200	25296	27496	1.47	16.86	18.33	1.07
1600	2200	26922	29122	1.38	16.83	18.20	1.00
1700	2200	28664	30864	1.29	16.86	18.16	0.94
1800	2200	30529	32729	1.22	16.96	18.18	0.88

$$* TFC = (P_k)K = (1) (2200) = 2200$$

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The simulated data are consistent with the apriori specifications of the designer. Increasing returns to the variable inputs occur until the output elasticity, E , is 1.0 and average variable costs are minimized. In Table 1, where capital is fixed at 2000 units, average variable costs decline until a production rate of 1500. In Table 2, where capital is fixed at 2200 units, average variable costs decline until a production rate of 1600. After the production rate where AVC is minimized ($E=1.0$), decreasing returns to the variable inputs occur, the output elasticity, E , becomes less than one, and average variable costs begin to increase.

Average total costs, ATC, change in a manner consistent with changes in average variable costs, AVC, and average fixed costs, AFC. Specifically, the average total costs are "U" shaped with the minimum levels occurring at a production rate greater than the rate corresponding to the minimum AVC. In Table 1, ATC is minimized at a production rate of 1600 whereas AVC is minimized at a production rate of 1500. In Table 2, ATC is minimized at a production rate of 1700 whereas AVC is minimized at a production rate of 1600.

Economies of scale are also reflected in the cost structure. As capital is increased from 2000 to 2200 units, the minimum ATC declines from 27.80 \$/unit in Table 1 to 18.16 \$/unit in Table 2. The degree in which economies of scale occur in the simulated cost structure was determined by specifying the output elasticities, E , corresponding to the different levels of capital, K , and the capital elasticity value, $a7$.

The "dual" production function implied by the cost structure can be observed by applying Sheppard's lemma and using equation 22 to derive the input demand for labor. Given the same production rates (1200 to 1800), and changing the capital input from 2000 to 2200 units, Table 3 summarizes the results.

TABLE 3

The Dual Production Function implied by the Cost Structure

Output	K = 2000		K = 2200	
	Labor	API	Labor	API
1200	1041.5	1.152	673.13	1.783
1300	1113.2	1.168	715.51	1.817
1400	1189.8	1.177	760.89	1.840
1500	1271.7	1.179	809.49	1.853
1600	1359.5	1.177	861.53	1.857
1700	1453.4	1.170	917.26	1.853
1800	1554.1	1.158	976.95	1.842

The simulated production function for labor is observed to behave in a manner consistent with duality theory. The average product of the variable input (labor), API, is the "dual" of the average variable cost. When AVC decreases, the average product of labor increases. When AVC is at its minimum level, the average product of labor is at its maximum value. As the level of the fixed input, K , is increased from 2000 to 2200 units, the API increases in tandem with the decline in the AVC and ATC observed in Tables 1 and 2. In this way, economies of scale on the cost side are mirrored by increasing returns to scale in the production technology.

The second variable input, materials CM), was also simulated and shown to behave in a consistent manner with duality theory but the results are not shown for sake of brevity.

CONCLUSIONS

Students using business simulations are supposed to learn, experientially, how the "real world" functions. Consequently it is important for the algorithms within the simulation to reflect, as much as possible, the relationships observed by empirical studies of the business environment. Although the theoretical and empirical properties found in the literature are well known, quantifying these relationships in a simulation are not straight forward. The functional forms used in a simulation need to be flexible enough to model a wide range of cost and production relationships, while maintaining the characteristics of stability and consistency. The intent of this study is not only to present an approach for modeling the supply side of

the firm, but to encourage other simulation designers to share, to a greater extent, the way in which they have modeled their simulations. This type of research should help simulation users better understand the cause and effect relationships embodied within the "black box" of business simulations. A better understanding of simulations by users and designers can only help facilitate the growth, development, and use of business simulations and experiential learning.

Specifically this study has argued that the properties of duality theory need to be addressed when designing short-run cost and production functions. The characteristics of the cost structure embodied in a business simulation imply certain characteristics relating to the production technology. If these relationships are not carefully modeled, inconsistencies between production and cost structures may develop. A review of the literature has indicated that there are some common problems in the way in which contemporary business simulations have designed their cost and production relationships.

The methodology developed in this paper to model the cost structure and production technology of the firm possesses a number of desirable properties:

(1) Cost information is used first to model the firm, and then applied to derive the associated production technology. This reduces the need to collect both cost and production data to model the firm. Also, cost information is more accessible than production information, making data collection easier and quicker.

(2) The application of Sheppard's lemma guarantees the characteristics designed in the cost structure will be embodied, in a consistent fashion, to the production technology of the firm. This will help avoid some of the pitfalls found in prior studies relating to the design of contemporary business simulations.

(3) The short-run cost function relates total variable costs directly to variable factor prices, the production rate, and the level of the fixed factors (e.g. capital). Non-separability between variable and fixed factors is assumed, making variable costs also a function of the level of the fixed inputs. This approach allows for variable elasticities, increasing and decreasing returns, and economies and diseconomies of scale.

(4) The parameters of the multiplicative functional form are easy to estimate. Only a limited amount of data is required to design the characteristics of the cost and production system, as indicated by the numerical example in this paper. The functional form is flexible enough to model a wide range of cost and production functions.

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