

IS THE GOLD /PRAY SIMULATION DEMAND MODEL VALID AND IS IT REALLY ROBUST?

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ABSTRACT

The purpose of this paper is to evaluate the Gold/Model presented at ABSEL in 1983 to determine whether the model is really valid and “Robust” as claimed by the authors. The model was subjected to rigorous sensitivity analysis and examined in light of different theories of advertising. The assumption that a multiplicative model is superior to non multiplicative models was also examined. The rigorous analysis of this paper found a number of problems inherent in the Gold/Pray model.

In 1981, Gold and Pray presented a paper at ABSEL which received the Best Simulation Paper award. The paper was the first of its kind at ABSEL and seemed to present an unique and viable approach to the development of demand algorithms in business simulations. Several papers followed which were based on this demand model (Fisk, 1985, Lai, 2003).

Except for one paper, no one has critically evaluated this model nor presented reasons why the model might not be valid nor robust. The only other paper which evaluated this model was by Lambert and Lamber (1988). One of the major problems as will be seen is that the demand model proposed by Gold and Pray can be unrealistic and highly unpredictable. Also, the bell-shaped marketing and research and development functions inherent in the model may be difficult to defend.

The authors present what they call a multiplicative equation. It is of the form where, for example, $Y = A \times B \times C$. In their model, in one sense A is price, B is marketing and C is R & D.

Some of the problems associated with this type of model were recognized by Lambert (1980) prior to the introduction of the Gold/Pray Model. Lambert correctly identified multiplicative models as being compensatory. The exact Gold/Pray model is:

$$Q = g1P^{-(g2+g3P)} M^{(g4 - g5M)} R^{(g6-g7R)}$$

$$g1 = 2.34 \times 10^{-12}$$

$$g2 = .15$$

$$g3 = .01$$

$$g4 = .3.88$$

$$g5 = .0000015$$

$$g6 = 3.88$$

$$g7 = .0000015$$

P - Price
M - Marketing
R - Research and development

FIRST PROBLEM: THE G/P MODEL DOES NOT ALLOW ADVERTISING OR R & D TO BE ZERO

The first problem concerns the effect of marketing (e.g., advertising) on demand. If advertising and R& D in the G/P model are zero, then demand is zero. There is no demand when price stands alone without advertising and R & D. For example, if price is \$50 and advertising and R & D are zero, then the value generated for demand is .00000000000050642052. Secondly, if advertising is \$50,000 and RD is 0, then demand is still zero.

Now strangely enough, economic textbooks are almost totally silent regarding advertising. However, it is never assumed that some advertising and research and development are required to avoid zero demand. Now we have a model being presented that requires some advertising and R & D, otherwise, there is no demand.

In the G/P model, demand based on price alone is always a very small number. This makes the model difficult to comprehend. An extremely small number generated by price and very large positive numbers for marketing and R & D seems a bit unusual. That the price portion of the equation, $P^{-(g2+g3P)}$, is always a very small number can be illustrated as follows: Assume that advertising and R & D both equal \$1,000. The following values are created by the Gold/Pray equation:

Advertising = \$1,000

R & D = \$1,000

See Table 1

On the other hand, when advertising is \$50,000 and RD is \$50,000 demand turns out to be an enormously large number, 2,770,456,430,311,750,000. It is hard to imagine a scenario where this quantity of demand could be considered reasonable.

It is obvious, therefore, that the components of the equation generates numbers that standing alone allow no common sense understanding. The only value of importance, therefore, is the net effect of the interaction of the three variables, price, advertising and R & D.

Table 1

| Price | Price Values | Advertising Values | R & D Values |
|-------|------------------------|--------------------|------------------|
| 100 | .00000000000001172778 | 1432016267618,142 | 432016267618.142 |
| 50 | .000000000000018402861 | 432016167618.142 | 432016167618.142 |
| 25 | .000000000000064571348 | 432016167618.142 | 432016167618.142 |

SECOND PROBLEM: THE GOLD/PRAY MODEL ALLOWS FOR ONLY ONE KIND OF SHIFT IN THE DEMAND CURVE

Virtually all economists agree that advertising shifts the demand curve. The question is: how do marketing variables such as advertising shift the demand curve? Most books do not say how nor do they give an illustration. One exception is Albrecht (1974):

Albrecht says, “The aim of advertising is both to shift to the right the demand curve faced by the individual and to make it less elastic.” (p. 531). His illustration is shown in Figure 1 below.

The Gold/pray model does not allow a change in the Y-intercept. The Y-intercept is the price value on the Y axis where demand is zero. There is no explanation nor an illustration in the paper of how their model shifts the demand curve. While the paper is unclear as to the nature of the shift, by using sensitivity analysis the nature of the shift can be determined. By holding price and R & D, constant, and the using different values for advertising, we discover

the nature of the shift as shown in Figure 2.

In the G/P model, the Y-intercept remains the same for all demands curves regardless of the degree of shift to the right. Albrecht, as can be seen, in his illustration(Figure 1), has advertising causing an increase in the Y-intercept and a decrease in the slope of the demand curve. The difference in the shift created by the G/P model and the shift illustrated in Figure 1 is too significant not to be noticed and explained.

The Gold/Pray model does not allow this kind of shift illustrated by Albrecht. Is a stationary Y-intercept with only change in the line slope a better definition of the effect of advertising? If so, then the burden of proof is on the authors to show that their concept of how advertising shifts the demand curve is correct or at least explain under what circumstances the demand curve shift in their model is valid. Also, in the G/P model, the elasticity of demand remains the same at each price whereas in the Figure 1 the elasticity of demand is less at each price. In the Albrecht graph of a shift in demand, the elasticity of demand has changed at each price. Whether or not elasticity of demand changes because of a shift in the demand curve is also very

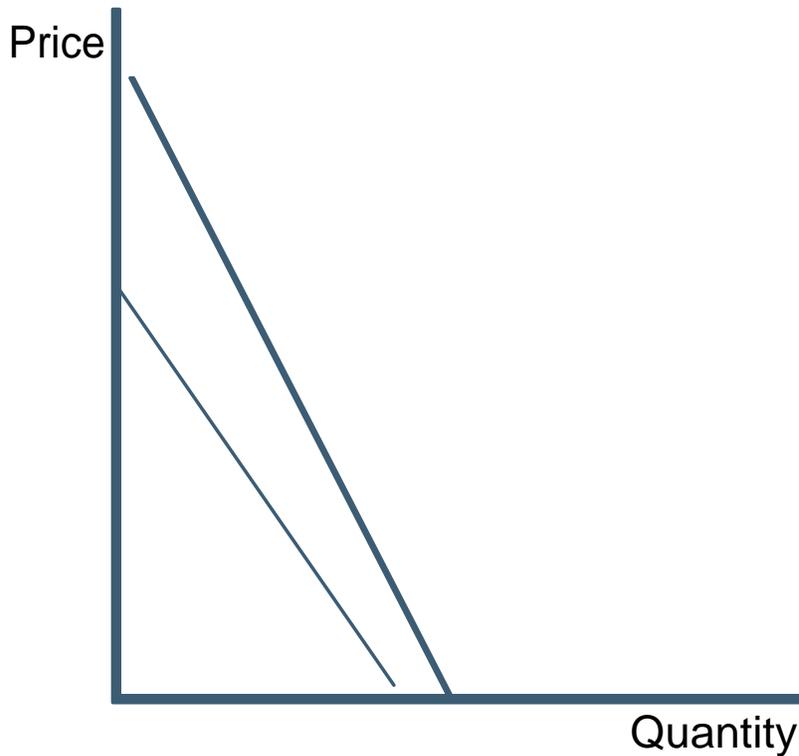


Figure 1

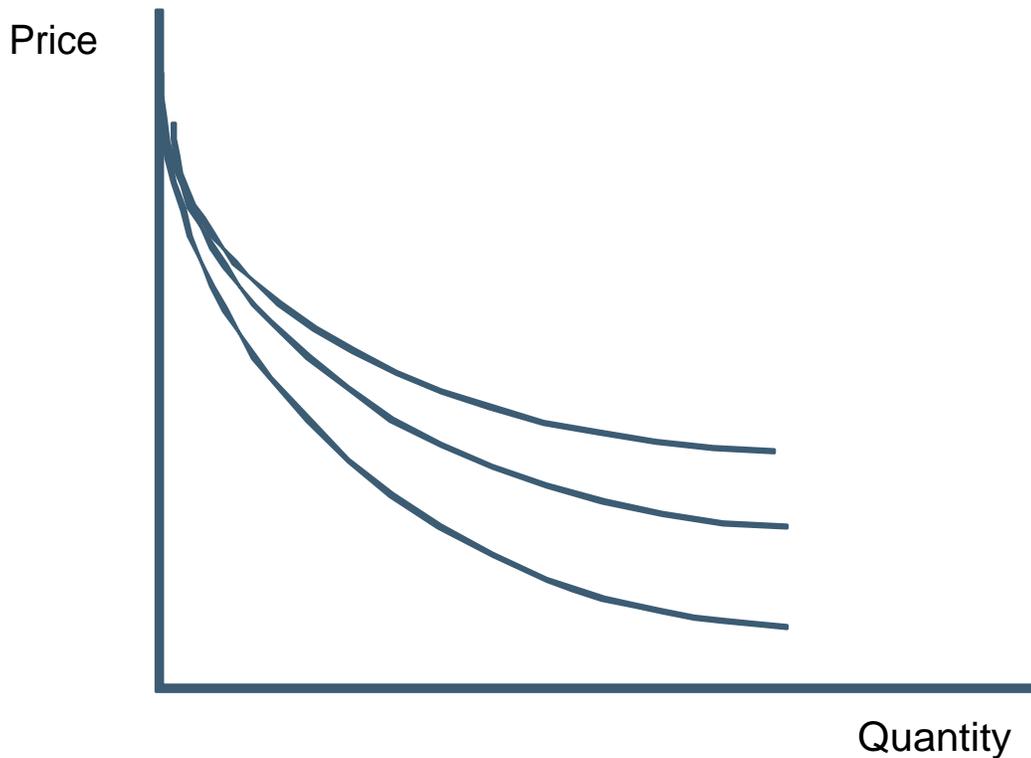


Figure 2

important and should require considerable attention by the game designer.

THIRD PROBLEM: POTENTIALLY CREATES UNREALISTICALLY LARGE DEMAND

Gold and Pray state that the model they presented is “Robust”. If by “Robust” the authors mean that a small change in marketing and R & D expenditures can cause an explosive increase in demand, then the authors are correct. For example assume the following:

| | |
|-----------|---------|
| Price | \$50 |
| Marketing | \$1000 |
| R & D | \$1,000 |

Given these values, the G/P model gives demand as 94,844,312. Now if we double the amounts for marketing and R & D:

| | |
|-----------|---------|
| Price | \$50 |
| Marketing | \$2,000 |
| R & D | \$2,000 |

Note: Gold and Pray do not provide parameters values for R & D nor illustrate the effect of changing values for R & D. For simplicity purposes in this paper, the same values assigned to marketing were assigned to R & D. For

marketing, the assigned values for g4 and g5 were 3.88 and .0000015 respectively. For R & D, the same values have been assigned: Consequently, values used here for g6 and g7 are 3.88 and .0000015 respectively.

Given these new values, the G/P model then shows demand to 10,068,595,959. A relatively small increase in marketing and R & D has caused an enormous increase in demand. A 100% increase in advertising has caused a 1,060 % increase in demand. It seems unreasonable that a modest \$4,000 expenditure for marketing and R & D could generate a demand that exceeds 10 billion units. If we let marketing and R & D be equal to \$50,000 each, then the demand that results is 7,623,901,410,962,230,000. In the G/P model and using the same parameters used by the authors, a very modest increase in marketing and R & D can cause an astronomical increase in demand.

The explosive power of a multiplicative model using the P/G model can be illustrated as follows:: Please See Table 2.

The numbers in the above table were generated by using the G/P model and using the parameters in their paper. Notice that the only values changing are in the advertising column. The column to observe carefully is the advertising column.

When advertising was \$100 the “A” factor was 57,504,257. However, when advertising is increased to \$1,000, there a 10 fold is increase in the advertising factor. However, notice what has happened to the “A” factor when advertising has increased to \$1,000. The A factor has become 432,016,167,168. Remarkably this is a 7,412 fold

Table 2

| Demand | Price Factor | Marketing Factor (Advertising) | R & D Factor |
|------------|-----------------------|--------------------------------|---------------|
| 1,675 | P = \$50 | Adv. = \$100 | R&D = \$100 |
| | .00000000000050642052 | 57,504,257 | 57,504,257 |
| | Price= \$50 | Adv. = \$200 | R & D = \$100 |
| 8,951 | .00000000000050642052 | 845,872,775 | 57,504,257 |
| | Price = \$50 | Adv. = \$300 | R & D = \$100 |
| 43,122 | .00000000000050642052 | 4,074,888,819 | 57,504,257 |
| | Price = \$50 | Adv = \$1,000 | R = \$100 |
| 12,580,887 | .00000000000050642052 | 432,016,167,168 | 57,504,257 |

increase. The Gold/Pray multiplicative model is driven by linear equations that serve as exponents. It should not be surprising then that a multiplicative model can have an explosive exponential effect. The multiplicative G/P model increases demand exponentially. Eventually, if advertising is increased enough the exponential increase will be staggering.

Whether this problem of an explosive demand potential is an inherent flaw in the model or simply the result of a poor choice of parameters is at this point not clear. Now it may be possible with some appropriate changes in the parameters to achieve realistic numbers, but given the prescribed parameters in the paper, the results are highly unrealistic. If the authors want to convince the serious reader and simulation designers that this model is both realistic and robust, the authors are obligated, I believe, to select or find realistic demand parameters values that demonstrate a reasonable relationship between marketing expenditures and increases in demand. The same is would be true for R & D parameters.

FOURTH PROBLEM: THE MODEL FOR BOTH MARKETING AND ADVERTISING CREATES BELL-SHAPED FUNCTIONS FOR MARKETING AND R & D.

An important issue concerns the nature or shape of the marketing function when price and R & D are held constant while different values are assigned to marketing (advertising). Through sensitivity analysis we discover that the marketing and R&D functions takes the form as shown in Figure 3. The G/P model has the advertising function for both advertising and R&D to be bell shaped more or less. The fact was revealed by Lambert and Lambert (1988)

At a certain level of advertising, demand begins to decrease. The same results happens when R&D is a marketing variable. Perhaps excessive advertising might upset customers to the point that they buy less, but it is hard to see why excessive R & D would eventually cause a decrease in sales. It might cause a decrease in net income but not a decrease in units sold.

When marketing variables other than price are introduced into the demand algorithm,

the question as to the nature of the marketing function and R & D functions arises. These functions theoretically may take on one of four shapes.

A linear function is shown in Figure 5 while a curvilinear function is shown in Figures 6 and 7.

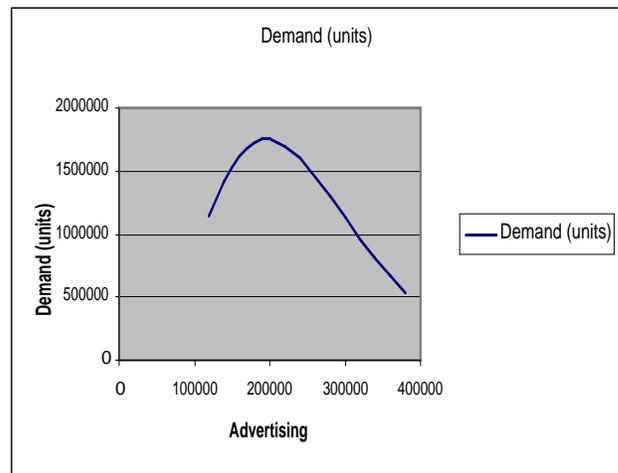


Figure 3

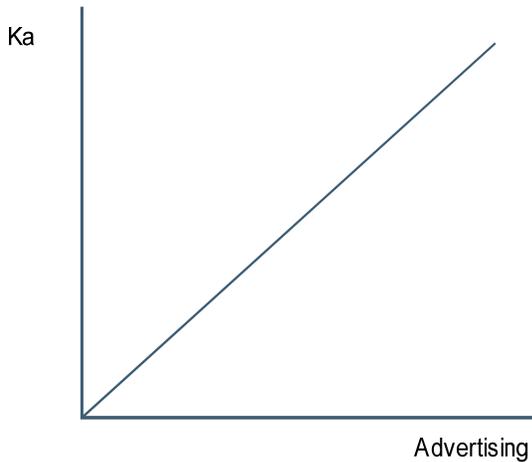


Figure 4

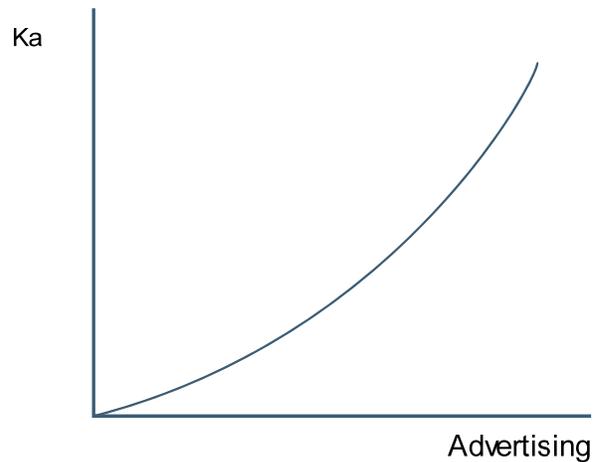


Figure 5

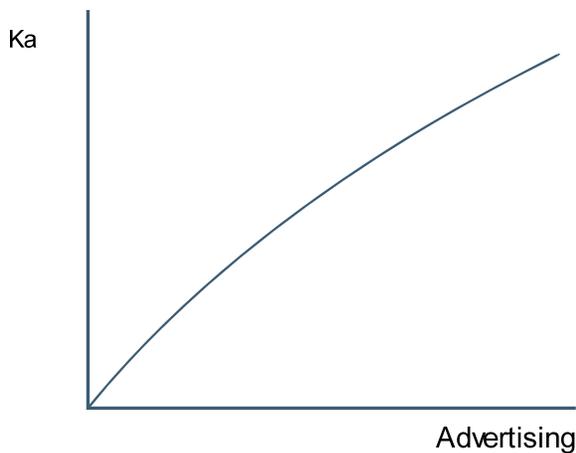


Figure 6

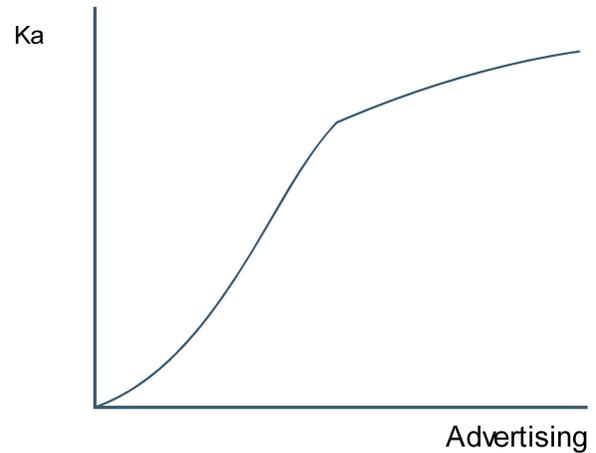


Figure 7

Figure 5 shows increasing returns while Figure 6 shows decreasing returns. The G/P model supports none of the functions illustrated above. The G/P model actually creates a marketing function that is bell-shaped. In Figure 7 is illustrated a marketing function that takes the form of a S-shaped curve. Of the four functions shown above, the general consensus is that a S-shaped function is the most valid.

The G/P model is complex mathematically. The difficulty in managing of parameters to get the right results suggests that either the equation is beyond the ability of the average simulation designer to use. Even if the equation were easy to use, there is still some doubt that the results are valid. The authors need to present convincing evidence that this model is valid and, if valid, to use examples that are realistic and meaningful. As pointed out by Lambert and Lambert (1988), a bell shaped function for advertising has questionable validity.

FIFTH PROBLEM: LACK OF PROOF THAT THE MULTIPLICATIVE MODEL IS DESIRABLE OR SUPERIOR

Gold and Pray describe their model as being multiplicative. The article by Gold and Pray in which their model was presented does not present any rationale as to why the multiplicative model is better. The article simply assumes that it is better.

The nature of this type of model needs to be understood. The meaning of a multiplicative model can be explained as follows: When the effect of one variable is multiplied by another variable in the equation, this effect may be referred to as multiplicative. As previously explained, the G/P equation takes the form of where $Y = A \times B \times C$.

The G/P model has three decision variables: price, marketing and research and development. However, the authors only illustrate two of three variables: price and

marketing. When the number of variables becomes two or greater additional problems are created. The dramatic effect of a change in price in the Gold/Pray model depends primarily on the interaction of these two decision variables.

Assume that at a given price of \$80 demand is 3,000 units. Let us call this value base demand.

Now let us assume that advertising will shift the demand curve by 20%. Q (demand) then would be: $Q = B^Q + B^Q(A\%) = B^Q(1+.2) = 3,000 \times 1.2 = 3,600$

Now assume that an expenditure of \$3,000 for R & D will increase demand by 30%. Also, for the moment let us assume advertising is zero.

$$Q = B^Q + (B^Q)R\% = B^Q(1 + R\%) = 3,000(1 + .3) = 3,900$$

It appears that advertising alone will increase demand by 600 units and R & D alone by 900 units. Now suppose in the same period both marketing and R & D respectively are \$1,000 such that the percentage increases in demand again are .2 and .3.

Now the multiplicative demand equation would be:

$$Q = B^Q(1 + A\%)(1 + R\%) = 3,000(1 + .2)(1 + .3) = 4,680$$

Rather than an 1,500 unit increase we have a 1,680 unit increase.

It might seem reasonable to argue that after the increase in advertising which increases demand to 3,600 that the increase in demand based on R & D should be based on the 3,600 rather than the original demand of 3,000. The argument in favor of a multiplicative equation is that it creates an interaction between marketing and R & D.

However, it could be argued that R & D does not shift the demand curve but keeps the demand curve more or less the same. Without R & D and, therefore, without new and improved products, demand might greatly decrease. It does not necessarily follow that R & D increases demand. It perhaps could be argued that it simply maintains demand and that ever increasing R & D expenditures are necessary to keep demand relatively stable. However, how advertising affects demand and, consequently, sales is an open question. Depending on circumstances, a number of different theories may be valid.

The problem with the G/P multiplicative model is that if parameters are not carefully selected and tested, the interaction of the demand variables may be potentially explosive even at some modest level of expenditures for the marketing and R & D decision variables. To create a demand model with three or more demand decisions that can control this potentially explosive effect may be extremely difficult or perhaps even impossible. Many business simulations have as many as five demand variables and the potential interaction of this many variables needs to be carefully analyzed.

SIXTH PROBLEM: NO ADJUSTMENT FOR THE NUMBER OF FIRMS IN COMPUTING INDUSTRY DEMAND

As presented in their 1983 paper, Gold and Pray did not adjust industry demand for the number of firms assigned to the industry by the user of the simulation. However, in the model for computing firm demand, the number of firms were taken into account and did affect the allocation of industry demand. If industry demand is not adjusted for the number of firms in the industry, then as the number of firms increase the size of allocated demand to each firm decreases. The implication by Gold and Pray is that the size of industry demand is independent of the number of firms. The basic question at issue then is: Does industry (market) demand remain the same as the number of firms increase?

In a business simulation used in a classroom environment, the number of firms is critical to the comparison and evaluation of performance. For example, let us assume that professor X has two classes and that in class 1 there are enough students for four teams but in class two there are enough students for eight teams. For illustration purposes, let us assume that both classes make identical decisions. If the computation of industry demand is not adjusted for the number of firms, then each team in class 1 will receive considerably less allocated industry demand. When a pie is divided among eight individuals rather than four, the slices become considerably smaller.

There is a need for the authors to explain how the model can be modified to take into account a difference in the number of firms in the industry.

A LOOK AT THE MULTIPLICATIVE MODEL MATHEMATICALLY

The Gold/Pray model at first glance is rather intimidating mathematically. However, in order to understand its exponentially explosive nature, a mathematical look inside the model is necessary. A more mathematical explanation of a multiplicative model can be given as follows:

The model requires a beginning value for price and quantity. To simplify matters for the moment let's use a linear definition of demand: $Q = (P_0 - P)/K$. We will now label Q as the base quantity; that is: $B^Q = Q$, where P_0 and K are starting parameter values and serve as the frame of reference of future changes in advertising and R & D. Now let us make the reasonable assumption that changes in advertising cause a percentage shift in the demand curve and that the A% is the value for the percentage change caused by advertising and B% is the value change in demand caused by R & D.

Based only on a change in advertising (for the moment let's assume R & D is zero) the new demand equation for Q would be:

$$Q = B^Q + A\%(B^Q) = B^Q(1 + A\%)$$

Now what if R & D also increases in the same time period? How does an increase in R & D affect demand given that advertising has also increased? The multiplicative approach first computes the demand shift based on the assumption that first shift is caused by advertising.

The next shift is caused by R & D and is computed based on the new demand curve resulting from the advertising expenditure. This is the approach, in essence, taken by Gold and Pray. Consequently, Gold and Pray would now compute demand as follows:

$$Q = B^Q + A\%(B^Q) + R\%(B^Q + A\%(B^Q))$$

$$Q = B^Q + A\%(B^Q) + R\%(B^Q) + RA\%(B^Q)$$

$$Q = B^Q(1 + A\% + R\% + R\%A\%)$$

While a bit difficult at first to see, the interactive term in the above equation is $R\%(A\%)$.

For the moment, let us assume the advertising causes a 20% increase in demand and that

R and D causes a 30% shift. Let us assume that initial price is \$80 and the $K = .01$ while P_0 (the Y-intercept) equals 110. Base demand therefore would be 3,000. Now given the percentage shifts cause by advertising and R & D, we have:

$$Q = 3,000(1 + .2 + .3 + .2(.3)) = 3,000(1.56) = 4,680$$

A non multiplicative approach would have been to define Q as follows:

$$Q = BQ + A\%(BQ) + R\%(BQ) = BQ(1 + A\% + R\%)$$

$$Q = 3,000(1.5) = 4,500$$

The term $R\%(A\%)$ does not appear in the last equation. Now the difference is clear. The multiplicative approach has one additional term: $R\%(A\%)$ which in this example causes demand to be \$180 larger. The non multiplicative equation does not contain the term $A\%(R\%)$.

To advocate that the multiplicative approach is superior to a non multiplicative model requires some evidence that the additional term, $R\%(A\%)$ is necessary. To do this, first of all, would require an argument that there are in essence two shifts that take place, a shift first by advertising and then a second shift by R & D based on the new demand curve caused by advertising. Some compelling evidence is required to prove that this is true in all cases. In absence of compelling evidence or arguments, the non multiplicative

equation for causing a shift in the demand curve seems just as valid and perhaps even more valid in some instances.

SUMMARY

In a simulation, the moment the game designer creates a demand algorithm which includes marketing variables such as advertising and R & D, the designer has imposed in the simulation a theory of advertising that may or may not have general acceptance. Various theories of advertising abound so the game designer may face a difficult challenge to select one specific theory. Hopefully, this would not happen, but it is possible for the game designer to inadvertently create a demand model involving advertising that can not be defended. The game designer may not recognize that his or her demand algorithm has serious flaws. Any demand algorithm involving two or more marketing variables must be subjected to rigorous testing.

If the game designer employs the Gold/Pray model or theory, the game designer has placed within the simulation a theory of advertising that may not be valid or may not achieved the results actually desired. The Gold/Pray model:

1. Does not allow for a change in the Y-Intercept because of an increase in advertising.
2. Does not allow the elasticity of demand to change with a change, for example, in advertising.
3. Creates bell-shaped marketing and R & D functions that many would question to be desirable or valid.
4. Has the potential to be excessively explosive in creating demand at a certain level of marketing and R & D expenditures.
5. Does not allow marketing or R & D expenditures to be zero. Some level of expenditure is required for both variables otherwise demand is zero.
6. Is a multiplicative model that may not be superior to a non multiplicative model.
7. For the market demand equation, there is no adjustment for the number of firms in the industry.

The first prerequisite for developing business simulation demand algorithm which includes advertising as a demand variable is an accepted or established theory of advertising. It is clear from the literature that a single accepted theory concerning advertising does not exist. It appears that some of our business simulations now in use that involve advertising in the demand algorithm may lack validity. Simulation designers need to disclose their demand algorithms so that they may be evaluated and tested for validity.

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